Progressive Science Initiative®

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When a cord or rope pulls on an object, it is said to be under tension, and the force it exerts on the object is called a tension force, $F_T$.

The forces on the right are not an action reaction pair as they both act on the bucket.

If the pail is moving up (or down) with a constant velocity, then $a_y = 0$.

\[ \Sigma F_y = F_T - mg = ma_y = 0 \]

\[ F_T = mg \]
17 A rope affixed to the ceiling is holding a bucket of water of mass 22.4 kg. What is the Tension force in the rope? Use $g = 10.0 \text{ m/s}^2$.

A 44.8 N  
B 448 N  
C 224 N  
D 22.4 N
18 A rope is tied to a bucket of water of mass 22.4 kg. The bucket is pulled upwards with an acceleration of 2.77 m/s$^2$. What is the Tension force in the rope? Use g = 10.0 m/s$^2$. 
Static Equilibrium

There is a whole field of problems in engineering and physics called "Statics" that has to do with cases where no acceleration occurs and objects remain at rest.

Anytime we construct bridges, buildings or houses, we want them to remain stationary, which is only possible if there is no acceleration or no net force.

There are two types of motion that we need to consider (and in both cases, motion is to be prevented!).

What are they?
Static Equilibrium

The two types of static equilibrium relate to linear (translational) and rotational acceleration.

Linear acceleration would be when the object, or components of the object, is moving in a straight line and rotational acceleration is when it pivots about a point and rotates. Neither works well in a building or a bridge.

In order to prevent acceleration (movement), for the first case, the net force is equal to zero, and in the second case, the net torque is zero. Only the linear acceleration will be covered in this section. Rotational equilibrium is covered in the Rotational Motion chapter.
Previous problems involved a rope supporting an object by exerting a vertical force straight upwards, along the same axis as the force $mg$ that was pulling it down. That led to the simplest case that if the bucket is moving up or down with a constant velocity, then $a_y = 0$, and the tension force, $F_T = mg$. 
There was also the case where an applied force acted on one object, but not the other. A Tension force acted between the two masses. Again, these forces acted in the same dimension.

Let's try a few problems on this again - it was covered earlier in this chapter, but it's a good time to review.
48 A box of mass 60.0 kg is suspended from a massless rope in an elevator that is moving up, but is slowing down with an acceleration of 2.20 m/s\(^2\). What is the tension in the rope? Use \(g = 10.0\) m/s\(^2\).
Multi-Correct: What horizontal forces are acting on the two blocks below?

A  Tension force on Block \( m_1 \)

B  Tension force on Block \( m_2 \)

C  Applied force on Block \( m_1 \)

D  Applied force on Block \( m_2 \)
A system of two blocks of masses 6.0 kg and 4.0 kg is accelerated by an applied force of 20.0 N on a frictionless horizontal surface. Draw a free body diagram for each block and find the Tension in the rope connecting the blocks.
Tension Force

A more interesting problem is for two (or more) ropes to support a stationary object \( (a = 0) \) by exerting forces at angles.

Since we're going to focus on the Tension force for a while, and using additional subscripts (1 and 2), we'll save notation and replace \( F_T \) with \( T \). As a physics person, you're allowed to do this!

In this case, since the it is at rest, the \( \Sigma F_x \) and \( \Sigma F_y \) on the object are zero.

Where would you see this outside the physics classroom?
Tension Force

In the cables that hold traffic lights over the street.
Since the only other force on the object is gravity, the vertical components of the force exerted by each rope must add up to mg. And if the object isn't moving, then $a_y = 0$.

$$\Sigma F_y = T_{1y} + T_{2y} - mg = ma_y = 0$$

$$T_{1y} + T_{2y} - mg = 0$$

$$T_{1y} + T_{2y} = mg$$
Tension Force

The only forces in the x direction are those that are provided by the x components of $T_1$ and $T_2$. Again, the object isn't moving, so $a_x = 0$.

$$\Sigma F_x = -T_{1x} + T_{2x} = ma_x = 0$$

$$-T_{1x} + T_{2x} = 0$$

$$T_{1x} = T_{2x}$$

*Note the negative sign on $T_{1x}$ since it is pointing to the left (negative x axis).*
Tension Force

Going forward, we will be dealing with the magnitudes of the various Tension components. We take into account the directions in the below equations, where $T_{1x}$ was assigned a negative value as it was pointing to the left, and $T_{2x}$, $T_{1y}$ and $T_{2y}$ were all assigned positive values.

That's the great value of making a sketch and a free body diagram - you don't have to worry about angles greater than $90^0$ and the different signs of the cosine and sine functions in the various graph quadrants.

$$T_{1y} + T_{2y} = mg$$
$$T_{1x} = T_{2x}$$
In the case of two ropes holding up a stationary bucket, what is the relationship of the magnitudes of the x components of $T_1$ and $T_2$?

A $T_{1x} = T_{2x}$

B $T_{1x}$ is greater than $T_{2x}$

C $T_{1x}$ is less than $T_{2x}$

D $T_{1x} = T_{2x} = 0$
In the case of two ropes holding up a stationary bucket, what is the relationship between $mg$ and the magnitudes of the $y$ components of $T_1$ and $T_2$?

A  $T_{1y} = T_{2y} + mg$

B  $T_{1y} = T_{2y} - mg$

C  $T_{1y} + T_{2y} = mg$

D  $T_{1y} = T_{2y} = mg$
Tension Force

$T_1$ and $T_2$ will now be resolved along the x and y axes. $mg$ is already just in the negative y axis direction.
Tension Force

Be careful of how the problem is stated. Sometimes the angles $\alpha_1$ and $\alpha_2$ - the angles that the ropes make with the support platform (ceiling, for example) - are given. They are complementary to the angles $\theta$ that are used here.
Time for a problem.

Calculate the tension in the two ropes if the first, $T_1$, is at an angle of $50^\circ$ from the vertical and the second, $T_2$, is at an angle of $20^\circ$ from the vertical and they are supporting an 8.0 kg mass.

Note we are using $\theta_1$ and $\theta_2$ and assume two significant figures for each angle.
Resolve $T_1$ into its x and y components:

$\sin \theta_1 = \frac{opp}{hyp}$

$\sin \theta_1 = \frac{T_{1x}}{T_1}$

$T_{1x} = T_1 \sin \theta_1$

$\cos \theta_1 = \frac{adj}{hyp}$

$\cos \theta_1 = \frac{T_{1y}}{T_1}$

$T_{1y} = T_1 \cos \theta_1$
Tension Force

Resolve $T_2$ into its x and y components:

$$\cos \theta_2 = \frac{adj}{hyp}$$

$$\cos \theta_2 = \frac{T_{2y}}{T_2}$$

$$T_{2y} = T_2 \cos \theta_2$$

$$\sin \theta_2 = \frac{opp}{hyp}$$

$$\sin \theta_2 = \frac{T_{2x}}{T_2}$$

$$T_{2x} = T_2 \sin \theta_2$$
Given that the Tension in rope 1 is $T_1 = 68 \text{ N}$, and $\theta_1 = 55^0$, find the x component of the Tension force.
Given that the Tension in rope 2 is \( T_2 = 79 \text{ N} \), and \( \theta_2 = 45^0 \), find the x component of the Tension force.
55 Given that the Tension in rope 1 is $T_1 = 68$ N, and $\theta_1 = 55^0$, find the y component of the Tension force.
Given that the Tension in rope 2 is $T_2 = 79$ N, and $\theta_2 = 45^0$, find the y component of the Tension force.
It's now time to take the resolved vectors, substitute in the given values and solve the simultaneous equations:

\[ \theta_1 = 50^\circ \]
\[ \theta_2 = 20^\circ \]
\[ m = 8.0 \]

\[ T_{1x} = T_1 \sin \theta_1 \quad T_{2x} = T_2 \sin \theta_2 \]
\[ T_{1y} = T_1 \cos \theta_1 \quad T_{2y} = T_2 \cos \theta_2 \]
\[ T_{1y} + T_{2y} = mg \]
\[ T_{1x} = T_{2x} \]
Tension Force

\[ T_{1x} = T_1 \sin \theta_1 = T_1 \sin(50^0) = .77T_1 \]
\[ T_{1y} = T_1 \cos \theta_1 = T_1 \cos(50^0) = .64T_1 \]
\[ T_{2x} = T_2 \sin \theta_2 = T_2 \sin(20^0) = .34T_2 \]
\[ T_{2y} = T_2 \cos \theta_2 = T_2 \cos(20^0) = .94T_2 \]
\[ mg = (8.0)(9.8) = 78 \]

Recall that the signs are already taken into account with the following two equations, so just substitute in the above values. *That's the advantage of a FBD and not worrying about the signs of the sine and cosine functions.*

\[ T_{1y} + T_{2y} = mg \quad .64T_1 + .94T_2 = 78 \]
\[ -T_{1x} + T_{2x} = 0 \]
\[ T_{1x} = T_{2x} \quad .77T_1 = .34T_2 \]
Tension Force

Equation 1: \[0.64T_1 + 0.94T_2 = 78\]

Equation 2: \[0.77T_1 = 0.34T_2\]

We now have two simultaneous equations with two variables. Solve Equation 2 for \(T_1\) and then substitute \(T_1\) into Equation 1 and solve for \(T_2\).

\[T_1 = \frac{0.34}{0.77}T_2 = 0.44T_2\]

\[0.64(0.44T_2) + 0.94T_2 = 78\]

\[T_2 = 64N\]

Now substitute \(T_2\) into Equation 2 and solve for \(T_1\).

\[0.77T_1 = 0.34(64)\]

\[T_1 = 28N\]
Tension Force

T₂ is greater than T₁, which means that it is picking up a greater "load." A way to determine this without the mathematics is to note that T₂ is more vertical than T₁.

So if two people are trying to lift a load - the stronger person should take the more vertical rope!

Also the sum of the magnitudes of T₁ and T₂ are greater than the weight of the box.
Tension Force

Take a limiting case where $\theta_1 = \theta_2 = \theta$. The ropes make equal angles with the vertical.

$$T_{1x} = T_1 \sin \theta_1 = T_1 \sin \theta$$
$$T_{2x} = T_2 \sin \theta_2 = T_2 \sin \theta$$

Since $T_{1x} = T_{2x}$

$$T_1 \sin \theta = T_2 \sin \theta$$
$$T_1 = T_2 = T$$

Each rope has the same tension - the load is shared equally.
Now, examine the forces in the y direction.

\[ T_{1y} = T_1 \cos \theta_1 = T \cos \theta \]
\[ T_{2y} = T_2 \cos \theta_2 = T \cos \theta \]

Since \( T_{1y} + T_{2y} = mg \)

\[ T \cos \theta + T \cos \theta = mg \]
\[ T = \frac{mg}{2 \cos \theta} \]

Let's take a limiting case again (physicists love doing this) - what happens as the support wires get more horizontal (\( \theta \) approaches \( 90^\circ \))? 
As $\theta$ approaches $90^0$ (the ropes become more horizontal), the Tension required to support the box approaches infinity.

Can you think of an example of this effect (and you can't use the traffic light one again)?
Tension Force

Electrical power transmission lines.

This helps explain why the lines sag - the force required to maintain a horizontal position would exceed the strength of the lines - and external wires are wrapped around them to add more support.
57 A lamp of mass m is suspended from two ropes of unequal length as shown below. Which of the following is true about the tensions $T_1$ and $T_2$ in the ropes?

A $T_1$ is less than $T_2$
B $T_1 = T_2$
C $T_1$ is greater than $T_2$
D $T_1 + T_2 = mg$
E $T_1 - T_2 = mg$
A mass $m$ is suspended from two massless strings of an equal length as shown below. The tension force in each string is:

A  $\frac{1}{2}mg\cos\theta$
B  $2\ mg\cos\theta$
C  $mg\cos\theta$
D  $mg/\cos\theta$
E  $\cos\theta/mg$