Residual: Difference between the observed (actual) value and the predicted (line of fit) value

Slope-Intercept Form of a linear function:  \( f(x) = mx + b \)

Forms of quadratic functions:

- Vertex Form:  \( f(x) = a(x-h)^2 + k \)
- Standard Form:  \( f(x) = ax^2 + bx + c \)
- Factored Form:  \( f(x) = (x-d)(x-e) \)

Quadratic Formula: If \( ax^2 + bx + c = 0 \), then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Zero-Product Property: If \( ab = 0 \), then \( a = 0 \) or \( b = 0 \)

Pythagorean Theorem:

In a right triangle, \( a^2 + b^2 = c^2 \), where \( a \) and \( b \) are the lengths of the legs, and \( c \) is the length of the hypotenuse.

Distance between two points \((x_1, y_1)\) and \((x_2, y_2)\):  \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

Slope of a line containing two points \((x_1, y_1)\) and \((x_2, y_2)\):  \( \frac{y_2 - y_1}{x_2 - x_1} \)
Unit 4, Topic 1

For items 1 through 3, classify each function below as linear, exponential, or quadratic. Justify your answers.

1. \[ \begin{array}{c|c}
   x & y \\
   \hline
   1 & 2 \\
   2 & 5 \\
   3 & 8 \\
   4 & 11 \\
\end{array} \]

2. \[ \begin{array}{c|c}
   x & y \\
   \hline
   1 & 2 \\
   2 & 5 \\
   3 & 10 \\
   4 & 17 \\
\end{array} \]

3. \[ \begin{array}{c|c}
   x & y \\
   \hline
   1 & 2 \\
   2 & 4 \\
   3 & 8 \\
   4 & 16 \\
\end{array} \]

4. A rectangle has a length that is 6 inches longer than its width. If \( w \) represents the width, write an expression, in terms of \( w \), for the area of the rectangle.

5. A rectangle has a length that is 4 inches shorter than its width. If \( w \) represents the width, write an expression, in terms of \( w \), for the area of the rectangle.
For items 6 through 8, a function of time is given. In each item, determine the average rate of change on the given interval. Give the units for your answer.

6.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>125</td>
</tr>
</tbody>
</table>

Average speed on the interval $2 \leq t \leq 5$:

Units:

7.

Average rate of bacteria growth on the interval $1 \leq t \leq 6$:

Units:
8. \( d(t) = 500t + t^2 \)

\( t \) in minutes, \( d(t) \) in kilometers

Average speed on the interval \( 0 \leq t \leq 4 \):

Units:

9. The number of weeds on a lawn can be written as the function \( W(d) = 25d + 60 \), where \( d \) represents the number of days since spring started.

a. Is the rate of change of this function constant? Explain your answer.

b. What is the rate of change?

c. What are the units for the rate of change?
11. Look at the pattern of dots below.

![Pattern of dots](image)

Let $D(n)$ be a function that represents the number of dots in figure number $n$.

a. Complete the table below.

<table>
<thead>
<tr>
<th>Figure number ($n$)</th>
<th>Number of Dots $D(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

b. Which type of function: linear, quadratic, or exponential, is represented by $D(n)$? Give a reason for your answer.

c. Write the function rule for $D(n)$.

d. How many dots will be in Figure 9? Show how you determined your answer.
19. The graph below represents \( y = x^2 \).

Look at the graphs, labeled I, II, III, and IV below.

Match the equations listed below with the graphs I, II, III, and IV above.

a. _____ \( y = \frac{-1}{3}x^2 \)  
   b. _____ \( y = 4x^2 \)

   c. _____ \( y = -5x^2 \)  
   d. _____ \( y = \frac{1}{2}x^2 \)
26. \( f(x) = 2(x-3)^2 - 8 \)

For items 27 through 30, add or subtract as indicated.

27. \((-3x^2 + 4x + 7) + (7x^2 - 3x - 12)\)

28. \((2x^2 - 5x - 9) - (4x^2 - 7x + 12)\)

29. \((x^2 - 4x + 11) + (8x^2 + 11x + 15)\)

30. \((4x^2 - 9x - 12) - (2x^2 + 9x - 11)\)
For items 31 through 36, multiply as indicated.

31. \((2x - 5)(4x + 7)\)

32. \((3x - 2)(x - 9)\)

33. \((2x - 3)^2\)

34. \((5x + 3)^2\)

35. \((x + 7)(x - 7)\)

36. \((3x - 5)(3x + 5)\)
For each rectangle below, determine the perimeter and the area. The area should be written as a trinomial.

37. \[ \text{Perimeter} \quad \text{Area as a trinomial} \]
38. \[ \text{Perimeter} \quad \text{Area as a trinomial} \]

Factor.

39. \[ x^2 - 8x - 20 \]
40. \[ x^2 + 16x + 60 \]
41. \[ x^2 - 36 \]
42. \[ x^2 - 121 \]
43. \[ 3x^2 - 5x - 2 \]
44. \[ 2x^2 - 10x - 12 \]
45. \[ -3x^2 - 21x - 36 \]
Unit 4, Topic 3

For items 46 through 48, use your calculator to graph the function, then state the number of real zeros.

46. \( f(x) = 6 - x^2 \)

47. \( f(x) = x^2 + x + 5 \)

48. \( f(x) = x^2 - 2x + 1 \)

49. Jack kicked a football. The height, \( h(t) \) in feet, of the ball after \( t \) seconds is given by the quadratic function \( h(t) = -16t^2 + 64t \).

   a. After how many seconds does the ball hit the ground? Show how you determined your answer.

   b. Does the ball reach its maximum height at \( t = 2 \) seconds? Show how you determined your answer.
50. An animal makes a leap into the air. A function for the height \( h(t) \), in meters, of the animal above the ground after \( t \) seconds is given by \( h(t) = -5t^2 + 12.5t \).

a. How long is the animal in the air? Show how you determined your answer.

b. What is the animal’s maximum height? Show how you determined your answer.
51. In your own words, what is the zero-product property?

For items 52 through 57, solve the equation using any method.

52. \((x - 7)^2 = 81\)  
53. \((x - 5)(3x - 2) = 0\)

54. \(7x^2 - 8x + 1 = 0\)  
55. \(x^2 - 10x - 1 = 0\)

56. \(x^2 + 6x = -8\)  
57. \(x^2 - 7 = 9\)
58. A farmer plants apple trees. The total number of apples can be represented by the function \( A(t) = 300t - 5t^2 \), where \( t \) is the number of trees that he plants.

a. Sketch the graph of \( A(t) = 300t - 5t^2 \) on the coordinate plane below. Show the points where \( t = 0, 10, 20, 30, 40, 50, 60 \).

b. The farmer wants to be able to harvest at least 4000 apples.

Use your graph above to solve the inequality \( 300t - 5t^2 \geq 4000 \).

c. What is the meaning of your solution in the context of this situation?
For items 60 and 61, determine the side of the right triangle marked $x$.

60. 

61. 

On the coordinate plane below, three line segments are shown, with their coordinates.

For items 62 and 63, find the distance between each pair of points.

62. $(7, -2)$ and $(-6, 3)$

63. $(-6, 3)$ and $(2, 9)$
66. Look at a scatterplot and the line of best fit below.

A residual for a value of $x$ is the difference between the observed (actual) value of $y$ and the predicted (line of fit) value of $y$.

a. What is the residual when $x = 3$?

b. What is the residual when $x = 6$?

c. What is the residual when $x = 9$?

67. a. In the 1980’s it was found that the correlation coefficient between the amount of ice cream sold and the number of crimes in New York City was $r = +0.97$. Does this mean that an increase in ice cream sales causes an increase in the number of crimes? Explain your reasoning.

b. Give another example of two variables that might have a high correlation coefficient, but the change in one of the variables does NOT cause a change in the other.
69. A carnival game kept track of the ages and what type of prize, if any, that people won at the game. The results are shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Children</th>
<th>Adults</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Prize</td>
<td>24</td>
<td>56</td>
<td>80</td>
</tr>
<tr>
<td>Small Prize</td>
<td>18</td>
<td>82</td>
<td>100</td>
</tr>
<tr>
<td>Big Prize</td>
<td>8</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>50</strong></td>
<td><strong>150</strong></td>
<td><strong>200</strong></td>
</tr>
</tbody>
</table>

a. Of the children who played, what fraction won a big prize?

b. What fraction of the small prize winners were adults?

c. What fraction of the total number of players were children?

d. The percent of children who won a small prize was (less than/equal to/greater than) the percent of adults who won no prize.
70. Montgomery County middle and high school students were asked if school should start later in the day. Here is some information about the responses.

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle School</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High School</td>
<td></td>
<td>225</td>
<td>250</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>350</td>
</tr>
</tbody>
</table>

a. Complete the table.

b. What fraction of high school students said yes?

c. Of the students who said no, what fraction were middle school students?

d. Of all of the students, what fraction were high school students who said no?
Unit 5

71. Which graph below represents the function \( f(x) = |x + 3| \)?

A

B

C

D
72. Jill takes a Sunday drive in her car. She drives at an average speed of 50 miles per hour for 3 hours. She takes a dinner break for 2 hours. After dinner, she travels at an average speed of 25 miles per hour for the next 4 hours.

Let \( t \) represent the number of hours since she started. Let \( D(t) \) represent the total distance that she has travelled after \( t \) hours.

a. What is the domain of \( t \)?

b. Sketch a graph of \( D(t) \) on the coordinate axes below.

![Graph of \( D(t) \)](image)

c. Write a piecewise function for \( D(t) \).

d. What was Jill’s average speed on the interval \( 0 \leq t \leq 9 \)? Show how you determined your answer.
Area/Circumference

Triangle: \[ A = \frac{1}{2}bh \]
Rectangle: \[ A = bh \]
Trapezoid: \[ A = \frac{1}{2}(b_1 + b_2)h \]
Parallellogram: \[ A = bh \]
Regular Polygon: \[ A = \frac{1}{2} \times \text{apothem} \times \text{perimeter} \]

Circle Area: \[ A = \pi r^2 \]
Circle Circumference: \[ C = 2\pi r = \pi d \]

Volume

Prism/Cylinder: \[ V = Bh = \text{area of base} \times \text{height} \]
Pyramid/Cone: \[ V = \frac{1}{3}Bh = \frac{1}{3} \times \text{area of base} \times \text{height} \]
Sphere: \[ V = \frac{4}{3}\pi r^3 \]

Density Formula

Density = \frac{\text{Mass}}{\text{Volume}}

Coordinate Geometry

Slope: \[ \frac{y_2 - y_1}{x_2 - x_1} \]
Midpoint: \[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
Distance: \[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
Conic Sections

Equation of a circle with center at $(h,k)$ and radius $r$: $(x-h)^2 + (y-k)^2 = r^2$

Equations of a parabola with vertex at the origin, with $p$ the distance from the vertex to the focus and vertex to directrix:

- $x^2 = 4py$ or $y = \frac{1}{4p}x^2$; opens up if $p > 0$, opens down if $p < 0$
- $y^2 = 4px$ or $x = \frac{1}{4p}y^2$; opens right if $p > 0$, opens left if $p < 0$

Circles

Arc Length (degrees): $S = \frac{x}{360}(2\pi r)$  
Arc Length (radians): $S = r\theta$

Sector Area (degrees): $A = \frac{x}{360}(\pi r^2)$  
Sector Area (radians): $A = \frac{\theta}{2\pi}(\pi r^2)$

Angle and Arc Formulas

- $m\angle A = \frac{1}{2}m\overline{BC}$
- $m\angle GDE = \frac{1}{2}(m\overline{GE} + m\overline{FH})$
- $m\angle J = \frac{1}{2}(m\overline{MN} - m\overline{LK})$

1 radian = $\frac{180}{\pi}$ degrees  
1 degree = $\frac{\pi}{180}$ radians
14. A company is producing a special part for a machine. The part consists of a cylinder of tin (white) that is inside of another cylinder made of copper (shaded). The part is shown below.

![Diagram](image)

a. What is the total volume of the entire part? You may give your answer in terms of $\pi$ or to the nearest cubic millimeter.

b. What is the volume of the tin used in the part? You may give your answer in terms of $\pi$ or to the nearest cubic millimeter.

c. What is the volume of copper used in the part? You may give your answer in terms of $\pi$ or to the nearest cubic millimeter.
Look at the graph of line $\overline{AB}$ on the coordinate plane below.

31. What is the length of $\overline{AB}$?

32. Point $C$ is between points $A$ and $B$ and has coordinates $(2, 2)$. What is the ratio $AC : CB$?

33. Determine the coordinates of point $D$ so that the ratio $AD : DB = 1:4$. 