I. Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ</td>
<td>change in quantity</td>
</tr>
<tr>
<td>±</td>
<td>plus or minus a quantity</td>
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<tr>
<td>∝</td>
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<td>=</td>
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<tr>
<td>≈</td>
<td>is approximately equal to</td>
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<tr>
<td>≰</td>
<td>is greater than or equal to</td>
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<tr>
<td>≪</td>
<td>is much less than</td>
</tr>
<tr>
<td>≫</td>
<td>is much greater than</td>
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</tbody>
</table>

<table>
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<th>Expression</th>
<th>Example</th>
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<tr>
<td>≡</td>
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II. Measurements and Significant Figures

Connecting Math to Physics  Math is the language of physics. Physicists use mathematical equations to describe relationships among the measurements that they make. Each measurement is associated with a symbol that is used in physics equations. These symbols are called variables.

Significant Figures

All measured quantities are approximate and have significant figures. The number of significant figures indicates the precision of the measurement. Precision is a measure of exactness. The number of significant figures in a measurement depends on the smallest unit on the measuring tool. The digit farthest to the right in a measurement is estimated.

Example: In the figure below, what is the estimated digit for each of the measuring sticks used to measure the length of the rod?

By using the lower measuring tool, the length is between 9 and 10 cm. The measurement would be estimated to the nearest tenth of a centimeter. If the length was exactly on the 9-cm or 10-cm mark, record it as 9.0 cm or 10.0 cm.

By using the upper measuring tool, the length is between 9.5 and 9.6 cm. The measurement would be estimated to the nearest hundredth of a centimeter. If the length was exactly on the 9.5-cm or 9.6-cm mark, record it as 9.50 cm or 9.60 cm.
All nonzero digits in a measurement are significant figures. Some zeros are significant, and some are not. All digits between and including the first nonzero digit from the left through the significant figure on the right are significant. Use the following rules when determining the number of significant figures.

1. Nonzero digits are significant.
2. Final zeros after a decimal point are significant.
3. Zeros between two significant figures are significant.
4. Zeros used only as placeholders are not significant.

**Example:** State the number of significant figures in each measurement.

- 5.0 g has two significant figures.  
- 14.90 g has four significant figures.  
- 0.0 has one significant figure.  
- 300.00 mm has five significant figures.  
- 5.06 s has three significant figures.  
- 304 s has three significant figures.  
- 0.0060 mm has two significant figures (6 and the last 0).  
- 140 mm has two significant figures (just 1 and 4).

**Practice Problems**

1. State the number of significant figures in each measurement.
   - a. 1405 m
   - b. 2.50 km
   - c. 0.0034 m
   - d. 12.007 kg
   - e. $5.8 \times 10^6$ kg
   - f. $3.03 \times 10^{25}$ mL

There are two cases in which numbers are considered exact and, thus, have an infinite number of significant figures.

1. Counting numbers have an infinite number of significant figures.
2. Conversion factors have an infinite number of significant figures.

**Examples:**

The factor “2” in $2 \text{mg}$ has an infinite number of significant figures.

*The number 2 is a counting number. It is an exact integer.*

The number “4” in 4 electrons has an infinite number of significant figures.

*Because you cannot have a partial electron, the number 4, a counting number, is considered to have an infinite number of significant figures.*

60 s/1 min has an infinite number of significant figures.

*There are exactly 60 seconds in 1 minute, thus there is an infinite number of significant figures in the conversion factor.*
Rounding

You can round a number to a specific place value (such as hundreds or tenths) or to a specific number of significant figures. To do this, determine the place to which you are rounding, and then use the following rules.

1. When the leftmost digit to be dropped is less than 5, that digit and any digits that follow are dropped. Then the last digit in the rounded number remains unchanged.
2. When the leftmost digit to be dropped is greater than 5, that digit and any digits that follow are dropped, and the last digit in the rounded number is increased by one.
3. When the leftmost digit to be dropped is 5 followed by a nonzero number, that digit and any digits that follow are dropped. The last digit in the rounded number increases by one.
4. If the digit to the right of the last significant digit is equal to 5 and 5 is followed by a zero or no other digits, look at the last significant digit. If it is odd, increase it by one; if it is even, do not round up.

Examples: Round the following numbers to the stated number of significant figures.

<table>
<thead>
<tr>
<th>Number</th>
<th>Rounded to</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.7645</td>
<td>8.76</td>
<td>1</td>
</tr>
<tr>
<td>8.7676</td>
<td>8.77</td>
<td>2</td>
</tr>
<tr>
<td>8.7519</td>
<td>8.8</td>
<td>3</td>
</tr>
<tr>
<td>92.350</td>
<td>92.4</td>
<td>4</td>
</tr>
<tr>
<td>92.25</td>
<td>92.2</td>
<td>4</td>
</tr>
</tbody>
</table>

PRACTICE PROBLEMS

2. Round each number to the number of significant figures shown in parentheses.
   a. 1405 m (2)
   b. 2.50 km (2)
   c. 0.0034 m (1)
   d. 12.007 kg (3)

Operations with Significant Figures

If using a calculator, do all of the operations with as much precision as the calculator allows, and then round the result to the correct number of significant figures. The number of significant figures in the result depends on the measurements and on the operation.

Addition and subtraction Round the result to the least precise value among the measurements—the smallest number of digits to the right of the decimal points.

Example: Add 1.456 m, 4.1 m, and 20.3 m.

The least precise values are 4.1 m and 20.3 m because they have only one digit to the right of the decimal points.

\[
\begin{align*}
1.456 \text{ m} & \quad \text{Add the numbers.} \\
4.1 \text{ m} & \\
+ 20.3 \text{ m} & \\
25.856 \text{ m} \\
25.9 \text{ m} & \quad \text{Round the result to place value of the least precise value.}
\end{align*}
\]
Multiplication and division  Look at the number of significant figures in each measurement. Perform the calculation. Round the result so that it has the same number of significant figures as the measurement with the least number of significant figures.

Example: Multiply 20.1 m by 3.6 m.

\[(20.1 \text{ m})(3.6 \text{ m}) = 72.36 \text{ m}^2\]

The least precise value is 3.6 m with two significant figures. The product can only have as many digits as the least precise of the multiplied numbers.

\[72 \text{ m} \quad \text{Round the result to two significant figures.}\]

### PRACTICE PROBLEMS

3. Simplify the following expressions using the correct number of significant figures.

   a. 5.012 km + 3.4 km + 2.33 km
   b. 45 g − 8.3 g
   c. 3.40 cm × 7.125 cm
   d. 54 m ÷ 6.5 s

Combination  When doing a calculation that requires a combination of addition/subtraction and multiplication/division, use the multiplication/division rule.

Examples:

\[x = 19 \text{ m} + (25.0 \text{ m/s})(2.50 \text{ s}) + \frac{1}{2}(-10.0 \text{ m/s}^2)(2.50 \text{ s})^2\]

\[= 5.0 \times 10^1 \text{ m} \quad \text{19 m only has two significant figures, so the answer should only have two significant figures.}\]

\[\text{slope} = \frac{70.0 \text{ m} - 10.0 \text{ m}}{29 \text{ s} - 11 \text{ s}}\]

\[= 3.3 \text{ m/s} \quad \text{29 s and 11 s only have two significant figures each, so the answer should only have two significant figures.}\]

Multistep calculations  Do not round to significant figures in the middle of a multistep calculation. Instead, round to a reasonable number of decimal places that will not cause you to lose significance in your answer. When you get to your final step where you are solving for the answer asked for in the question, you should then round to the correct number of significant figures.

Example:

\[F = \sqrt{(24 \text{ N})^2 + (36 \text{ N})^2}\]

\[= \sqrt{576 \text{ N}^2 + 1296 \text{ N}^2} \quad \text{Do not round to 580 N and 1300 N.}\]

\[= \sqrt{1872 \text{ N}^2} \quad \text{Do not round to 1800 N.}\]

\[= 43 \text{ N} \quad \text{Final answer, so it should be rounded to two significant figures.}\]

If you had rounded in each step, you would have obtained an answer of 44 N. This might seem like a small discrepancy, but in more complex calculations, the effects can be large. For example, if you calculate \(5 \times 5 \times 5 \times 5 \times 5\) without rounding your answer will be 3125, which rounds to 3000. If you round at each step, however, the answer you find is 2500, which rounds to 2000.
III. Fractions, Ratios, Rates, and Proportions

Fractions
A fraction names a part of a whole or a part of a group. It also can express a ratio (see page 840). A fraction consists of a numerator, a division bar, and a denominator.

\[
\frac{\text{numerator}}{\text{denominator}} = \frac{\text{number of parts chosen}}{\text{total number of parts}}
\]

Simplification Sometimes, it is easier to simplify an expression before substituting the known values of the variables. Variables often cancel out of the expression.

Example: Simplify \( \frac{pn}{pw} \).

\[
\frac{pn}{pw} = \left( \frac{p}{p} \right) \left( \frac{n}{w} \right)
\]

Factor out the \( p \) in the numerator and the denominator, and break the fraction into the product of two fractions.

\[
= (1) \left( \frac{n}{w} \right)
\]

Substitute \( \frac{p}{p} = 1 \).

\[
= \frac{n}{w}
\]

Multiplication and division To multiply fractions, multiply the numerators and multiply the denominators.

Example: Multiply the fractions \( \frac{5}{a} \) and \( \frac{t}{b} \).

\[
\left( \frac{5}{a} \right) \left( \frac{t}{b} \right) = \frac{5t}{ab}
\]

Multiply the numerators and the denominators.

To divide fractions, multiply the first fraction by the reciprocal of the second fraction. To find the reciprocal of a fraction, invert it—switch the numerator and the denominator.

Example: Divide the fraction \( \frac{5}{a} \) by \( \frac{t}{b} \).

\[
\frac{5}{a} \div \frac{t}{b} = \left( \frac{5}{a} \right) \left( \frac{b}{t} \right)
\]

Multiply the first fraction by the reciprocal of the second fraction.

\[
= \frac{5b}{at}
\]

Multiply the numerators and the denominators.

Addition and subtraction To add or subtract two fractions, first write them as fractions with a common denominator. To do this, multiply each fraction by the denominator of the other in the form of a fraction equal to one. Then add or subtract the numerators.

Example: Add the fractions \( \frac{1}{a} \) and \( \frac{2}{b} \).

\[
\frac{1}{a} + \frac{2}{b} = \left( \frac{1}{a} \right) \left( \frac{b}{b} \right) + \left( \frac{2}{b} \right) \left( \frac{a}{a} \right)
\]

Multiply each fraction by a fraction equal to 1.

\[
= \frac{b}{ab} + \frac{2a}{ab}
\]

Multiply the numerators and the denominators.

\[
= \frac{b + 2a}{ab}
\]

Write a single fraction with the common denominator.

PRACTICE PROBLEMS

4. Perform the indicated operation. Write the answer in simplest form.

a. \( \frac{1}{x} + \frac{y}{3} \)

b. \( \frac{a}{2b} - \frac{3}{b} \)

c. \( \frac{3}{x} \left( \frac{1}{y} \right) \)

d. \( \frac{2a}{5} \div \frac{1}{2} \)
**Ratios**

A ratio is a comparison between two numbers by division. Ratios can be written in several different ways. The ratio of 2 and 3 can be written in four different ways:

\[
\begin{align*}
&2 \text{ to } 3 \\
&2 \text{ out of } 3 \\
&2:3 \\
&\frac{2}{3}
\end{align*}
\]

**Rates**

A rate is a ratio that compares two quantities with different measurement units. A unit rate is a rate that has been simplified so that the denominator is 1.

**Example:** Write 98 km in 2.0 hours as a unit rate.

98 km in 2.0 hours is a ratio of \(\frac{98 \text{ km}}{2.0 \text{ hours}}\)

\[
\frac{98 \text{ km}}{2.0 \text{ hours}} = \left( \frac{98}{2.0} \right) \left( \frac{\text{km}}{\text{hour}} \right)
\]

Split the fraction into the product of a number fraction and a unit fraction.

\[
= (49) \left( \frac{\text{km}}{\text{hour}} \right)
\]

Simplify the number fraction.

\[
= 49 \text{ km per hour or } 49 \text{ km/h}
\]

**Example:** Write 16 Swedish crowns in 2 U.S. dollars as a unit rate.

16 Swedish crowns in $2.00 American currency is a ratio of \(\frac{16 \text{ Swedish crowns}}{2 \text{ U.S. dollars}}\)

\[
\frac{16 \text{ Swedish crowns}}{2 \text{ U.S. dollars}} = \left( \frac{16}{2} \right) \left( \frac{\text{Swedish crowns}}{\text{U.S. dollars}} \right)
\]

\[
= (8) \left( \frac{\text{Swedish crowns}}{\text{U.S. dollars}} \right)
\]

\[
= 8 \text{ Swedish crowns per U.S. dollar}
\]

or 8 Swedish crowns/U.S. dollar

**Proportions**

A proportion is an equation that states that two ratios are equal:

\[
\frac{a}{b} = \frac{c}{d}, \text{ where } b \text{ and } d \text{ are not zero.}
\]

Proportions used to solve ratio problems often include three numbers and one variable. You can solve the proportion to find the value of the variable. To solve a proportion, use cross multiplication.

**Example:** Solve the proportion \(\frac{a}{b} = \frac{c}{d}\) for \(b\).

Cross multiply: \(\frac{a}{b} = \frac{c}{d}\)

\[
bc = ad
\]

Write the equation resulting from cross multiplying.

\[
b = \frac{ad}{c}
\]

Solve for \(b\).

**PRACTICE PROBLEMS**

5. Solve the following proportions.

a. \(\frac{4}{x} = \frac{2}{3}\)

b. \(\frac{13}{15} = \frac{n}{75}\)

c. \(\frac{36}{12} = \frac{s}{16}\)

d. \(\frac{2.5}{5.0} = \frac{7.5}{w}\)
IV. Exponents, Powers, Roots, and Absolute Value

Exponents

An exponent is a number that tells how many times a number \((a)\) is used as a factor. An exponent is written as a superscript. In the term \(a^n\), \(a\) is the base and \(n\) is the exponent. \(a^n\) is called the \(n\)th power of \(a\) or \(a\) raised to the \(n\)th power.

Connecting Math to Physics  
A subscript is not an exponent. In physics, a subscript is used to further describe the variable. For example, \(v_0\) can be used to represent the velocity at time 0. A subscript is a part of the variable.

Positive exponent  
For any nonzero number \((a)\) and any integer \((n)\)

\[ a^n = (a)(a)\ldots(a) \text{  } n \text{  times} \]

Examples: Simplify the following exponent terms.

\[ 10^4 = (10)(10)(10)(10) = 10,000 \]
\[ 2^3 = (2)(2)(2) = 8 \]

Zero exponent  
For any nonzero number \((a)\)

\[ a^0 = 1 \]

Examples: Simplify the following zero exponent terms.

\[ 2^0 = 1 \quad 13^0 = 1 \]

Negative exponent  
For any nonzero number \((a)\) and any integer \((n)\)

\[ a^{-n} = \frac{1}{a^n} \]

Examples: Write the following negative exponent terms as fractions.

\[ 2^{-1} = \frac{1}{2} \quad 2^{-2} = \frac{1}{2^2} = \frac{1}{4} \]

Square and Cube Roots

A square root of a number is one of its two equal factors. A radical sign \((\sqrt{\text{ })}\) indicates a square root. A square root can be shown as exponent \(\frac{1}{2}\) as in \(\sqrt{b} = b^{\frac{1}{2}}\). You can use a calculator to find square roots.

Examples: Simplify the following square root terms.

\[ \sqrt{a^2} = \sqrt{(a)(a)} = a \]
\[ \sqrt{9} = \sqrt{(3)(3)} = 3 \]
\[ \sqrt{64} = \sqrt{(8.0)(8.0)} = 8.0 \]

Keep two significant figures.

\[ \sqrt{38.44} = 6.200 \]

Place two zeros to the right of the calculator answer to keep four significant figures.

\[ \sqrt{39} = 6.24497 \ldots = 6.2 \]

Round the answer to keep two significant figures.
A cube root of a number is one of its three equal factors. A radical sign with the number 3 (\(\sqrt[3]{\phantom{0}}\)) indicates a cube root. A cube root also can be shown as exponent \(\frac{1}{3}\), as in \(\sqrt[3]{b} = b^{\frac{1}{3}}\).

**Example:** Simplify the following cube root terms.

\[
\sqrt[3]{125} = \sqrt[3]{(5.00)(5.00)(5.00)} = 5.00 \\
\sqrt[3]{39.304} = 3.4000
\]

**PRACTICE PROBLEMS**

6. Find each root. Round the answer to the nearest hundredth.
   a. \(\sqrt[3]{22}\)
   b. \(\sqrt[3]{729}\)
   c. \(\sqrt[3]{676}\)
   d. \(\sqrt[3]{46.656}\)

7. Simplify by writing without a radical sign.
   a. \(\sqrt[3]{16a^2b^6}\)
   b. \(\sqrt{t^6}\)

8. Write using exponents.
   a. \(\sqrt{n^3}\)
   b. \(\frac{1}{\sqrt{a}}\)

**Operations with Exponents**

In the following operations with exponents, \(a, b, m,\) and \(n\) can be numbers or variables.

**Product of powers** To multiply terms with the same base, add the exponents, as in \((a^m)(a^n) = a^{m+n}\).

**Quotient of powers** To divide terms with the same base, subtract the bottom exponent from the top exponent, as in \(a^m/a^n = a^{m-n}\).

**Power of a power** To calculate the power of a power, use the same base and multiply the exponents, as in \((a^m)^n = a^{mn}\).

**\(n\)th root of a power** To calculate the root of a power, use the same base and divide the power exponent by the root exponent, as in \(\sqrt[n]{a^m} = a^{\frac{m}{n}}\).

**Power of a product** To calculate the power of a product of \(a\) and \(b\), raise both to the power and find their product, as in \((ab)^n = a^n b^n\).

**PRACTICE PROBLEMS**

9. Write an equivalent form using the properties of exponents.
   a. \(\frac{x^4 t^2}{x^3}\)
   b. \(\sqrt[3]{t^3}\)
   c. \((d^2 n)^2\)
   d. \(x^2 \sqrt{x}\)

10. Simplify \(\frac{m}{q} \sqrt{\frac{2qv}{m}}\).
**Absolute Value**

The absolute value of a number \( n \) is its magnitude, regardless of its sign. The absolute value of \( n \) is written as \(|n|\). Because magnitudes cannot be less than zero, absolute values always are greater than or equal to zero.

*Examples:*

\[
\begin{align*}
|3| &= 3 \\
|-3| &= 3
\end{align*}
\]

**V. Scientific Notation**

A number of the form \( a \times 10^n \) is written in scientific notation, where \( 1 \leq a \leq 10 \), and \( n \) is an integer. The base (10) is raised to a power \( (n) \).

**Connecting Math to Physics**

Physicists commonly use scientific notation to express measurements that are greater than 10 or less than 1. For example, the mass of a proton is written as \( 6.73 \times 10^{-28} \) kg. The density of water is written as \( 1.000 \times 10^3 \) kg/m\(^3\). This shows, using significant digit rules, that this measurement is exactly 1000 to four significant figures. However, writing the density of water as \( 1000 \) kg/m\(^3\) would imply that it has only one significant digit, which is incorrect. Scientific notation helps physicists keep accurate track of significant figures.

**Large Numbers—Using Positive Exponents**

Multiplying by a power of 10 is like moving the decimal point that same number of places to the right (if the power is positive) or to the left (if the power is negative). To express a large number in scientific notation, first determine the value for \( a \), \( 1 \leq a \leq 10 \). Count the number of decimal places from the decimal point in \( a \) to the decimal point in the number. Use that count as the power of 10. A calculator shows scientific notation with \( e \) for exponent, as in \( 2.4e + 11 = 2.4 \times 10^{11} \). Some calculators use an \( E \) to show the exponent, or there is often a place on the display where the calculator can show smaller-sized digits representing the exponent.

*Example: Write 7,530,000 in scientific notation.*

The value for \( a \) is 7.53. (The decimal point is to the right of the first nonzero digit.) So the form will be \( 7.53 \times 10^6 \).

\[7,530,000 = 7.53 \times 10^6\]  

There are six decimal places, so the power is 6.

To write the standard form of a number expressed in scientific notation, write the value of \( a \), and place extra zeros to the right of the number. Use the power and move the decimal point in \( a \) that many places to the right.

*Example: Write the following number in standard form.*

\[2.389 \times 10^5 = 2.38900 \times 10^5 = 238,900\]
Small Numbers—Using Negative Exponents

To express a small number in scientific notation, first determine the value for \(a\), \(1 \leq a \leq 10\). Then count the number of decimal places from the decimal point in \(a\) to the decimal point in the number. Use that number as the power of 10. Multiplying by a number with a negative power is the same as dividing by that number with the corresponding positive power.

**Example:** Write 0.000000285 in scientific notation.

The value for \(a\) is 2.85. (The decimal point is to the right of the first nonzero digit.) So the form will be \(2.85 \times 10^{-7}\). There are seven decimal places, so the power is \(-7\).

To express a small number in standard form, write the value for \(a\) and place extra zeros to the left of \(a\). Use the power and move the decimal point in \(a\) that many places to the left.

**Example:**

\[1.6 \times 10^{-4} = 0.00016\]

### PRACTICE PROBLEMS

11. Express each number in scientific notation.
   - a. 456,000,000
   - b. 0.000020

12. Express each number in standard notation.
   - a. \(3.03 \times 10^{-7}\)
   - b. \(9.7 \times 10^{10}\)

### Operations with Scientific Notation

Calculating with numbers written in scientific notation uses the properties of exponents.

**Multiplication** Multiply the terms, and add the powers of 10.

**Example:** Simplify.

\[(4.0 \times 10^{-6})(1.2 \times 10^{5}) = (4.0 \times 1.2)(10^{-6} \times 10^{5})\]

\[= (4.8)(10^{-6+5})\]

\[= (4.8)(10^{-1})\]

\[= 4.8 \times 10^{-3}\]

**Division** Divide the base numbers and subtract the exponents of 10.

**Example:** Simplify.

\[\frac{9.60 \times 10^{7}}{1.60 \times 10^{3}} = \frac{9.60}{1.60} \times \frac{10^{7}}{10^{3}}\]

\[= 6.00 \times 10^{7-3}\]

\[= 6.00 \times 10^{4}\]
Addition and subtraction  To add and subtract numbers in scientific notation, the powers of 10 must be the same. Thus, you may need to rewrite one of the numbers with a different power of 10. With equal powers of 10, use the distributive property.

Example: Simplify.

\[(3.2 \times 10^5) + (4.8 \times 10^5) = (3.2 + 4.8) \times 10^5\]

Group terms.

\[= 8.0 \times 10^5\]

Add terms.

Example: Simplify.

\[(3.2 \times 10^5) + (4.8 \times 10^4) = (3.2 \times 10^5) + (0.48 \times 10^5)\]

Rewrite \(4.8 \times 10^4\) as \(0.48 \times 10^5\).

\[= (3.2 + 0.48) \times 10^5\]

Group terms.

\[= 3.68 \times 10^5\]

Add terms.

Round using the addition significant figures rule.

### PRACTICE PROBLEMS

13. Evaluate each expression; express the result in scientific notation.

   a. \((5.2 \times 10^{-1}) (4.0 \times 10^8)\)

   b. \((2.4 \times 10^5) + (8.0 \times 10^4)\)

### VI. Equations

#### Order of Operations

Scientists and mathematicians have agreed on a set of steps or rules, called the order of operations, so that everyone interprets mathematical symbols in the same way. Follow these steps in order when you evaluate an expression or use a formula.

1. Simplify the expressions inside grouping symbols, such as parentheses ( ), brackets [ ], braces { }, and fraction bars.
2. Evaluate all powers and roots.
3. Do all multiplications and/or divisions from left to right.
4. Do all additions and/or subtractions from left to right.

A useful way to remember these is the phrase “Please excuse my dear Aunt Sally.” The first letter of each word represents an operation: parentheses, exponents, multiplication, division, addition, subtraction.

Example: Simplify the following expression.

\[4 + 3(4 - 1) - 2^3 = 4 + 3(3) - 2^3\]

Order of operations step 1.

\[= 4 + 3(3) - 8\]

Order of operations step 2.

\[= 4 + 9 - 8\]

Order of operations step 3.

\[= 5\]

Order of operations step 4.

Connecting Math to Physics  The previous example was shown step-by-step to demonstrate the order of operations. When solving a physics problem, do not round to the correct number of significant figures until after the final calculation.

In calculations involving an expression in a numerator and an expression in a denominator, the numerator and the denominator are separate groups and you should calculate them before dividing the numerator by the denominator. The multiplication/division rule for significant figures is used to determine the final number of significant figures.
Solving Equations
To solve an equation means to find the value of the variable that makes the equation a true statement. To solve equations, apply the distributive property and the properties of equality. Any properties of equalities that you apply on one side of an equation, you also must apply on the other side.

**Distributive property** For any numbers \(a, b,\) and \(c,\)

\[a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac\]

**Example:** Use the distributive property to expand the following expression.

\[3(x + 2) = 3x + (3)(2) = 3x + 6\]

**Addition and subtraction properties of equality** If two quantities are equal, and the same number is added to (or subtracted from) each, then the resulting quantities also are equal.

If \(a = b,\) then

\[a + c = b + c \quad \text{and} \quad a - c = b - c\]

**Example:** Solve \(x - 3 = 7\) using the addition property.

\[x - 3 = 7\]
\[x - 3 + 3 = 7 + 3\]
\[x = 10\]

**Example:** Solve \(t + 2 = -5\) using the subtraction property.

\[t + 2 = -5\]
\[t + 2 - 2 = -5 - 2\]
\[t = -7\]

**Multiplication and division properties of equality** If two equal quantities each are multiplied by (or divided by) the same number, then the resulting quantities also are equal.

If \(a = b,\) then

\[ac = bc\] and \[\frac{a}{c} = \frac{b}{c},\] for \(c \neq 0\)

**Example:** Solve \(\frac{1}{4}a = 3\) using the multiplication property.

\[\frac{1}{4}a = 3\]
\[\left(\frac{a}{4}\right)(4) = 3(4)\]
\[a = 12\]

**Example:** Solve \(6n = 18\) using the division property.

\[6n = 18\]
\[\frac{6n}{6} = \frac{18}{6}\]
\[n = 3\]

**Example:** Solve \(2t + 8 = 5t - 4\) for \(t.\)

\[2t + 8 = 5t - 4\]
\[8 + 4 = 5t - 2t\]
\[12 = 3t\]
\[4 = t\]
Isolating a Variable

Suppose an equation has more than one variable. To isolate a variable—that is, to solve the equation for a variable—write an equivalent equation so that one side contains only that variable with a coefficient of 1.

Connecting Math to Physics  Isolate the variable \( P \) (pressure) in the ideal gas law equation.

\[
\frac{PV}{V} = \frac{nRT}{V}
\]

Divide both sides by \( V \).

\[
P \left( \frac{V}{V} \right) = \frac{nRT}{V}
\]

Group \( \frac{V}{V} \).

\[
P = \frac{nRT}{V}
\]

Substitute \( \frac{V}{V} = 1 \).

### PRACTICE PROBLEMS

**14. Solve for \( x \).**

a. \( 2 + 3x = 17 \)

b. \( x - 4 = 2 - 3x \)

c. \( t - 1 = \frac{x + 4}{3} \)

d. \( a = \frac{b + x}{c} \)

e. \( \frac{2x + 3}{x} = 6 \)

f. \( ax + bx + c = d \)

Square Root Property

If \( a \) and \( n \) are real numbers, \( n > 0 \), and \( a^2 = n \), then \( a = \pm \sqrt{n} \).

Connecting Math to Physics  Solve for \( v \) in Newton’s second law for a satellite orbiting Earth.

\[
\frac{mv^2}{r^2} = \frac{Gm_{\text{Earth}}m}{r^2}
\]

Multiply both sides by \( r \).

\[
\frac{mv^2}{r} = Gm_{\text{Earth}}m
\]

Substitute \( \frac{r}{r} = 1 \).

\[
v^2 = \frac{Gm_{\text{Earth}}m}{m}
\]

Divide both sides by \( m \).

\[
v^2 = \frac{Gm_{\text{Earth}}}{r}
\]

Substitute \( \frac{m}{m} = 1 \).

\[
\sqrt{v^2} = \pm \sqrt{\frac{Gm_{\text{Earth}}}{r}}
\]

Take the square root.

\[
v = \sqrt{\frac{Gm_{\text{Earth}}}{r}}
\]

Use the positive value for speed.

When using the square root property, it is important to consider what you are solving for. Because we solved for speed in the above example, it did not make sense to use the negative value of the square root. Also, you need to consider if the negative or positive value gives you a realistic solution. For example, when using the square root property to solve for time, a negative value might give you a time before the situation even started.
Quadratic Equations

A quadratic equation has the form $ax^2 + bx + c = 0$, where $a \neq 0$. A quadratic equation has one variable with a power (exponent) of 2. It also might include that same variable to the first power. You can estimate the solutions by graphing on a graphing calculator. If $b = 0$, then there is no $x$-term in the quadratic equation. In this case, you can solve the equation by isolating the squared variable and finding the square root of each side of the equation using the square root property.

Quadratic Formula

You can find the solutions of any quadratic equation by using the quadratic formula. The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

As with the square root property, it is important to consider whether the solutions to the quadratic formula give you a realistic answer to the problem you are solving. Usually, you can throw out one of the solutions because it is unrealistic. Projectile motion often requires the use of the quadratic formula when solving equations, so keep the realism of the solution in mind when solving.

PRACTICE PROBLEMS

15. Solve for $x$.
   a. $4x^2 - 19 = 17$
   b. $12 - 3x^2 = -9$
   c. $x^2 - 2x - 24 = 0$
   d. $24x^2 - 14x - 6 = 0$

Dimensional Calculations

When doing calculations, you must include the units of each measurement that is written in the calculation. All operations that are performed on the number also are performed on its units.

Connecting Math to Physics

The acceleration due to gravity ($a$) is given by the equation $a = \frac{2\Delta x}{\Delta t^2}$. A free-falling object near the Moon drops 20.5 m in 5.00 s. Find the acceleration ($a$). Acceleration is measured in meters per second squared.

$$a = \frac{2\Delta x}{\Delta t^2}$$

$$a = \frac{2(20.5 \text{ m})}{(5.00 \text{ s})^2}$$

The number 2 is an exact number, so it does not affect the determination of significant figures.

Calculate and round to three significant figures.
Unit conversion  Use a conversion factor to convert from one measurement unit to another of the same type, such as from minutes to seconds. This is equivalent to multiplying by one.

Connecting Math to Physics  Find Δx when \( v_i = 67 \text{ m/s} \) and \( \Delta t = 5.0 \text{ min} \). Use the equation \( \Delta x = v_i \Delta t \).

\[
\frac{60 \text{ seconds}}{1 \text{ minute}} = 1
\]

\[
\Delta x = v_i \Delta t
\]

\[
\Delta x = \frac{67 \text{ m}}{\text{s}} \left( \frac{5.0 \text{ min}}{1 \text{ min}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right)
\]

\[
\Delta x = 20,100 \text{ m} = 2.0 \times 10^4 \text{ m}
\]

Multiply by the conversion factor \( \frac{60 \text{ s}}{1 \text{ min}} = 1 \)

Calculate and round to two significant figures. The numbers 60 s and 1 min are exact numbers, so they do not affect the determination of significant figures.

PRACTICE PROBLEMS

16. Simplify \( \Delta t = \frac{4.0 \times 10^2 \text{ m}}{16 \text{ m/s}} \).

17. Find the velocity of a dropped brick after 5.0 s using \( v = a \Delta t \) and \( a = -9.8 \text{ m/s}^2 \).

18. Calculate the product:

\[
\left( \frac{32 \text{ cm}}{1 \text{ s}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)
\]

19. An Olympian ran 100.0 m in 9.87 s. What was the speed in kilometers per hour?

Dimensional Analysis

Dimensional analysis is a method of doing algebra with the units. It often is used to check the validity of the units of a final result and the equation being used, without completely redoing the calculation.

Physics Example  Verify that the final answer of \( x_f = x_i + v_i t + \frac{1}{2} a t^2 \) will have the units meters (m).

\( x_i \) is measured in meters (m).

\( t \) is measured in seconds (s).

\( v_i \) is measured in meters per second (m/s).

\( a \) is measured in meters per second squared (m/s^2).

\[
x_f = m + \left( \frac{m}{s} \right)(s) + \frac{1}{2} \left( \frac{m}{s^2} \right) \left( \frac{s^2}{s^2} \right)
\]

Substitute the units for each variable.

\[
= m + m(1) + \frac{1}{2} (m)(1)
\]

Simplify the fractions using the distributive property.

\[
= m + m + \frac{1}{2} m
\]

Substitute \( s/s = 1, s^2/s^2 = 1 \).

Everything simplifies to m; thus \( x_e \) is in m.

The factor of \( \frac{1}{2} \) in the above does not apply to the units. It applies only to any number values that would be inserted for the variables in the equation. It is easiest to remove number factors such as the \( \frac{1}{2} \) when first setting up the dimensional analysis.
VII. Graphs of Relations

The Coordinate Plane
You can locate points on a plane in reference to two perpendicular number lines, called axes. The horizontal number line is called the x-axis and represents the independent variable. The vertical number line is called the y-axis and represents the dependent variable. A point is represented by two coordinates \((x, y)\), which also is called an ordered pair. The value of the independent variable \(x\) always is listed first in the ordered pair. The ordered pair \((0, 0)\) represents the origin.

Graphing Data to Determine a Relationship
Use the following steps to graph data.

1. Identify the independent and the dependent variables.
2. Draw two perpendicular axes. Label each axis using the variable names.
3. Determine the range of data for each variable. Use the ranges to decide on a convenient scale for each axis. Mark and number the scales.
4. Plot each data point.
5. If the points seem to lie approximately in a line, draw a best-fit line. The line should be as close to as many points as possible. If the points do not lie in a line, draw a smooth curve through as many points as possible. If there does not appear to be a trend, do not draw any line or curve.
6. Write a title that clearly describes what the graph represents.

<table>
<thead>
<tr>
<th>Applied Force (N)</th>
<th>Distance Stretched (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.8</td>
</tr>
<tr>
<td>20</td>
<td>2.1</td>
</tr>
<tr>
<td>30</td>
<td>2.8</td>
</tr>
<tr>
<td>40</td>
<td>4.2</td>
</tr>
<tr>
<td>50</td>
<td>4.7</td>
</tr>
</tbody>
</table>
**Interpolating and Extrapolating**

Interpolation is a process used to estimate a value for a relation that lies between two known values. Extrapolation is a process used to estimate a value for a relation that lies beyond the known values. The equation of the best-fit line helps you interpolate and extrapolate.

*Example:* Using the data and the graph, estimate how far the spring will stretch if a force of 25 N is applied. Use interpolation.

- Draw a best-fit line.
- Draw a line segment from 25 N on the x-axis to the best-fit line.
- Draw a line segment from that intersection point to the y-axis.
- Read the scale on the y-axis. A force of 25 N will stretch the spring 2.4 cm.

*Example:* Use extrapolation to estimate how far the spring will stretch if a 60-N force is applied.

- Draw a line segment from 60 on the x-axis to the best-fit line. Extend the best-fit line if necessary.
- Read the corresponding value on the y-axis. Extend the axis scale if necessary.
- A force of 60 N will stretch the spring 5.8 cm.

**Interpreting Line Graphs**

A line graph shows the linear relationship between two variables. Two types of line graphs that describe motion are used frequently in physics.

**Connecting Math to Physics**

This line graph shows a changing relationship between the two graphed variables. The line graph above shows a constant relationship between the two graphed variables.
Linear Equations

A linear equation can be written as a relation (or a function), $y = mx + b$, where $m$ and $b$ are real numbers, $m$ represents the slope of the line, and $b$ represents the intercept, the point at which the line crosses the $y$-axis.

The graph of a linear equation is a line. The line represents all of the solutions of the linear equation. To graph a linear equation, choose three values for the independent variable. (Only two points are needed, but the third point serves as a check.) Calculate the corresponding values for the dependent variable. Plot each ordered pair $(x, y)$ as a point. Draw a line through the points.

Example: Graph $y = -\frac{1}{2}x + 3$

Calculate three ordered pairs to obtain points to plot.

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Slope

The slope of a line is the ratio of the change in $y$-coordinates to the change in $x$-coordinates. It is the ratio of the vertical change (rise) to the horizontal change (run). This number tells you how steep the line is. It can be a positive number or a negative number.

To find the slope of a line, select two points, $(x_1, y_1)$ and $(x_2, y_2)$. Calculate the run, which is the difference (change) between the two $x$-coordinates, $x_2 - x_1 = \Delta x$. Calculate the rise, which is the difference (change) between the two $y$-coordinates, $y_2 - y_1 = \Delta y$. Form the ratio.

\[
\text{Slope } m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}
\]

where $x_1 \neq x_2$
Direct Variation

If there is some nonzero constant \((m)\) such that \(y = mx\), then \(y\) varies directly with \(x\). That means as the independent variable \(x\) doubles, the dependent variable \(y\) doubles. The variables \(x\) and \(y\) also are said to be proportional. This is a linear equation of the form \(y = mx + b\) in which the value of \(b\) is zero. The graph passes through the origin, \((0, 0)\).

**Connecting Math to Physics**  In the force equation for an ideal spring, \(F = -kx\), where \(F\) is the force on the spring, \(-k\) is the spring constant, and \(x\) is the spring's displacement, the force on the spring varies directly with (is proportional to) the spring's displacement. That is, the force on the spring increases as the spring's displacement increases.

Inverse Variation

If there is some nonzero constant \((m)\) such that \(y = \frac{m}{x}\), then \(y\) varies inversely with \(x\). That means as the independent variable \(x\) increases, the dependent variable \(y\) decreases. The variables \(x\) and \(y\) also are said to be inversely proportional. This is not a linear equation because it contains the product of two variables. The graph of an inverse relationship is a hyperbola. This relationship can be written as

\[
\begin{align*}
  y &= \frac{m}{x} \\
  y &= m \frac{1}{x} \\
  xy &= m
\end{align*}
\]

*Example:* Graph the equation \(y = \frac{90}{x}\).

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(y)</td>
</tr>
<tr>
<td>---</td>
<td>-----</td>
</tr>
<tr>
<td>-10</td>
<td>-9</td>
</tr>
<tr>
<td>-6</td>
<td>-15</td>
</tr>
<tr>
<td>-3</td>
<td>-30</td>
</tr>
<tr>
<td>-2</td>
<td>-45</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

**Connecting Math to Physics**  In the equation for the speed of a wave \((\lambda = \frac{v}{f})\) where \(\lambda\) is wavelength, \(f\) is frequency, and \(v\) is wave speed, wavelength varies inversely with (is inversely proportional to) frequency. That is, as the frequency of a wave increases, the wavelength decreases. \(v\) is constant.
Quadratic Graphs

A quadratic relationship is a relationship of the form

\[ y = ax^2 + bx + c, \text{ where } a \neq 0. \]

A quadratic relationship includes the square of the independent variable \( x \). The graph of a quadratic relationship is a parabola. Whether the parabola opens upward or downward depends on whether the value of the coefficient of the squared term \( a \) is positive or negative.

*Example:* Graph the equation \( y = -x^2 + 4x - 1 \).

### Ordered Pairs

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>-6</td>
</tr>
</tbody>
</table>

### Quadratic Equation Graph

Connecting Math to Physics  
A position-time graph in the shape of a quadratic relation means that the object is moving at a constant acceleration.

\[ x_i = 2 \text{ m} + (1 \text{ m/s})t + \frac{1}{2}(2 \text{ m/s}^2)t^2 \]
VIII. Geometry and Trigonometry

**Perimeter and Area**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter, Circumference</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Square</strong></td>
<td>$P = 4a$</td>
<td>$A = a^2$</td>
</tr>
<tr>
<td>side $a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Rectangle</strong></td>
<td>$P = 2l + 2w$</td>
<td>$A = lw$</td>
</tr>
<tr>
<td>length $l$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>width $w$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Triangle</strong></td>
<td>$P = \text{side 1 + side 2} + \text{side 3}$</td>
<td>$A = \frac{1}{2}bh$</td>
</tr>
<tr>
<td>base $b$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>height $h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Circle</strong></td>
<td>$C = 2\pi r$</td>
<td>$A = \pi r^2$</td>
</tr>
<tr>
<td>radius $r$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Surface Area and Volume**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Surface Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cube</strong></td>
<td>$SA = 6a^2$</td>
<td>$V = a^3$</td>
</tr>
<tr>
<td>side $a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cylinder</strong></td>
<td>$SA = 2\pi rh + 2\pi r^2$</td>
<td>$V = \pi r^2h$</td>
</tr>
<tr>
<td>radius $r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>height $h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sphere</strong></td>
<td>$SA = 4\pi r^2$</td>
<td>$V = \frac{4}{3}\pi r^3$</td>
</tr>
<tr>
<td>radius $r$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Connecting Math to Physics** Look for geometric shapes in your physics problems. They could be in the form of objects or spaces. For example, vectors can sometimes form two-dimensional shapes.

**Area Under a Graph**

To calculate the approximate area under a graph, cut the area into smaller pieces and find the area of each piece using the formulas shown above. To approximate the area under a line, cut the area into a rectangle and a triangle, as shown below on the left.

To approximate the area under a curve, draw several rectangles from the $x$-axis to the curve, as shown below on the right. Using more rectangles with a smaller base will provide a closer approximation of the area.

**Position v. Time**

Total area $= \text{Area of the rectangle} + \text{Area of the triangle}$

**Position v. Time**

Total area $= \text{Area 1} + \text{Area 2} + \text{Area 3} + \ldots$
Right Triangles

The Pythagorean theorem states that if $a$ and $b$ are the measures of the legs of a right triangle and $c$ is the measure of the hypotenuse, then

$$c^2 = a^2 + b^2.$$

To determine the length of the hypotenuse, use the square root property. Because distance is positive, the negative value does not have meaning.

$$c = \sqrt{a^2 + b^2}$$

Example: In the triangle, $a = 4$ cm and $b = 3$ cm. Find $c$.

$$c = \sqrt{a^2 + b^2}$$

$$= \sqrt{(4 \text{ cm})^2 + (3 \text{ cm})^2}$$

$$= \sqrt{16 \text{ cm}^2 + 9 \text{ cm}^2}$$

$$= \sqrt{25 \text{ cm}^2}$$

$$= 5 \text{ cm}$$

45°-45°-90° triangles The length of the hypotenuse is $\sqrt{2}$ multiplied by the length of a leg.

30°-60°-90° triangles The length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.
Trigonometric Ratios

A trigonometric ratio is a ratio of the lengths of sides of a right triangle. The most common trigonometric ratios are sine, cosine, and tangent. To memorize these ratios, learn the acronym SOH-CAH-TOA. SOH stands for Sine equals Opposite over Hypotenuse. CAH stands for Cosine equals Adjacent over Hypotenuse. TOA stands for Tangent equals Opposite over Adjacent.

<table>
<thead>
<tr>
<th>Words</th>
<th>Memory Aid</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>The sine is the ratio of the length of the side opposite to the angle over the length of the hypotenuse.</td>
<td>SOH [ \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} ]</td>
<td>[ \sin \theta = \frac{a}{c} ]</td>
</tr>
<tr>
<td>The cosine is the ratio of the length of the side adjacent to the angle over the length of the hypotenuse.</td>
<td>CAH [ \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} ]</td>
<td>[ \cos \theta = \frac{b}{c} ]</td>
</tr>
<tr>
<td>The tangent is the ratio of the length of the side opposite to the angle over the length of the side adjacent to the angle.</td>
<td>TOA [ \tan \theta = \frac{\text{opposite}}{\text{adjacent}} ]</td>
<td>[ \tan \theta = \frac{a}{b} ]</td>
</tr>
</tbody>
</table>

Example: In right triangle ABC, if \(a = 3\) cm, \(b = 4\) cm, and \(c = 5\) cm, find \(\sin \theta\) and \(\cos \theta\).

\[
\sin \theta = \frac{3\, \text{cm}}{5\, \text{cm}} = 0.6 \\
\cos \theta = \frac{4\, \text{cm}}{5\, \text{cm}} = 0.8
\]

Example: In right triangle ABC, if \(\theta = 30.0^\circ\) and \(c = 20.0\) cm, find \(a\) and \(b\).

\[
a = (20.0\, \text{cm})(\sin 30.0^\circ) = 10.0\, \text{cm} \\
b = (20.0\, \text{cm})(\cos 30.0^\circ) = 17.3\, \text{cm}
\]

Law of Cosines and Law of Sines

The laws of cosines and sines let you calculate the sides and angles of any triangle.

Law of cosines The law of cosines is the same as the Pythagorean theorem, except for the last term. \(\theta\) is the angle opposite side \(c\). If the angle \(\theta\) is \(90^\circ\), the \(\cos \theta = 0\) and the last term equals zero. If \(\theta\) is greater than \(90^\circ\), its cosine is a negative number.

\[
c^2 = a^2 + b^2 - 2ab \cos \theta
\]

Example: Find the length of the third side of a triangle with \(a = 10.0\) cm, \(b = 12.0\) cm, \(\theta = 110.0^\circ\).

\[
c^2 = a^2 + b^2 - 2ab \cos \theta \\
c = \sqrt{a^2 + b^2 - 2ab \cos \theta} \\
= \sqrt{(10.0\, \text{cm})^2 + (12.0\, \text{cm})^2 - 2(10.0\, \text{cm})(12.0\, \text{cm})(\cos 110.0^\circ)} \\
= \sqrt{1.00 \times 10^2\, \text{cm}^2 + 144\, \text{cm}^2 - (240.0\, \text{cm}^2)(\cos 110.0^\circ)} \\
= 18.1\, \text{cm}
\]
Law of sines  The law of sines is an equation of three ratios, where \(a\), \(b\), and \(c\) are the sides opposite angles \(A\), \(B\), and \(C\), respectively. Use the law of sines when you know the measures of two angles and any side of a triangle.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

Example: In a triangle, \(C = 60.0^\circ\), \(a = 4.0\) cm, \(c = 54.6\) cm. Find the measure of angle \(A\).

\[
\frac{\sin A}{a} = \frac{\sin C}{c}
\]

\[
\sin A = \frac{a \sin C}{c}
\]

\[
\sin A = \frac{(4.0\text{ cm})(\sin 60.0^\circ)}{4.6\text{ cm}}
\]

\[
A = 49^\circ
\]

Inverses of Sine, Cosine, and Tangent
The inverses of sine, cosine, and tangent allow you to do the reverse of the sine, cosine, and tangent functions and find the angle. The trigonometric functions and their inverses are as follows:

<table>
<thead>
<tr>
<th>Trigonometric Function</th>
<th>Inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = \sin x)</td>
<td>(x = \sin^{-1} y) or (x = \arcsin y)</td>
</tr>
<tr>
<td>(y = \cos x)</td>
<td>(x = \cos^{-1} y) or (x = \arccos y)</td>
</tr>
<tr>
<td>(y = \tan x)</td>
<td>(x = \tan^{-1} y) or (x = \arctan y)</td>
</tr>
</tbody>
</table>

Example: Solve \(0.62 = \sin \theta\) for \(\theta\).

\[
\theta = \sin^{-1} 0.62 = 38^\circ
\]

Graphs of Trigonometric Functions
The sine function \((y = \sin x)\) and the cosine function \((y = \cos x)\) are periodic functions. The period for each function is \(2\pi\). \(x\) can be any real number. \(y\) is a real numbers between \(-1\) and \(1\), inclusive.
IX. Logarithms

Logarithms with Base $b$

Let $b$ and $x$ be positive integers with $b \neq 1$. The logarithm of $x$ with base $b$, written $\log_b x$, equals $y$, where $y$ is the exponent that makes the equation $b^y = x$ true. The log to the base $b$ of $x$ is the number to which you can raise $b$ to get $x$.

$$\log_b x = y \quad \text{if and only if} \quad b^y = x$$

Memory aid: “the log is the exponent”

Examples: Calculate the following logarithms.

1. $\log_2 \frac{1}{16} = -4$  
   Because $2^{-4} = \frac{1}{16}$

2. $\log_{10} 1000 = 3$  
   Because $10^3 = 1000$

When you want to find the log of a number, you also can use a computer.

Connecting Math to Physics  Physicists use logarithms to work with measurements that extend over several orders of magnitude, or powers of 10. Geophysicists use the Richter scale, a logarithmic scale that allows them to rate earthquakes from 0 to 7, or larger, though the power of earthquakes differ by 7 or more powers of 10.

Common Logarithms

Base 10 logarithms are called common logarithms. They often are written without the subscript 10.

$$\log_{10} x = \log x \quad x > 0$$

Antilogarithms or Inverse Logarithms

An antilogarithm is the inverse of a logarithm. An antilogarithm is the number that has a given logarithm.

Examples: Solve $\log x = 4$ for $x$.

$$\log x = 4$$

$$x = 10^4 \quad 10^4 \text{ is the antilogarithm of } 4.$$ 

Connecting Math to Physics  An equation for loudness ($L$) in decibels is $L = 10 \log_{10} R$, where $R$ is the relative intensity of the sound. Calculate $R$ for a fireworks display with a loudness of 130 decibels.

130 = 10 $\log_{10} R$  

Divide by 10.

13 = $\log_{10} R$  

Use the logarithm rule.

$$R = 10^{13}$$

When you know the logarithm of a number and want to know the number itself, use a calculator to find the antilogarithm.

PRACTICE PROBLEMS

21. Write the following in exponential form: $\log_2 81 = 4$

22. Write the following in logarithmic form: $10^{-3} = 0.001$

23. Find $x$. $\log x = 3.125$