Progressive Science Initiative®

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Work and Energy

- Energy and the Work-Energy Theorem
- Forces and Potential Energy
- Conservation of Energy
- Power
Work and the Work-Energy Theorem
Conservation Principles

The most powerful concepts in science are called "conservation principles". These principles allow us to solve problems without worrying too much about the details of a process.

We just have to take a snapshot of a system initially and finally; by comparing those two snapshots we can learn a lot.
Conservation Principles

A good example is a bag of candy.

If you know that there are 50 pieces of candy at the beginning. And you know that none of the pieces have been taken out or added...you know that there must be 50 pieces at the end.
Conservation Principles

You can change the way you arrange them by moving them around...but you still will have 50 pieces. In that case we would say that the number of pieces of candy is conserved.

That is, we should always get the same amount, regardless of how they are arranged.
We also have to be clear about the system that we're talking about. If we're talking about a specific type of candy...we can't suddenly start talking about a different one and expect to get the same answers.

Conservation Principles

We must define the system whenever we use a conservation principle.
Energy is a conserved property of nature. It is not created or destroyed. Therefore in a closed system we will always have the same amount of energy.

The only way the energy of a system can change is if it is open to the outside...this means that energy has been added or taken away.
What is Energy?

It turns out that energy is so fundamental, like space and time, that there is no good answer to this question. However, just like space and time, that doesn't stop us from doing very useful calculations with energy.

We may not be able to define energy, but because it is a conserved property of nature, it's a very useful idea.
Conservation of Energy

If we call the amount of energy that we start with "$E_o$" and the amount we end up with as "$E_f$" then we would say that if no energy is added to or taken away from a system that

$$E_o = E_f$$

It turns out there are only two ways to change the energy of a system. One is with heat (which we won't deal with here) the other is with Work, "$W$".

If we define positive work as that work which increases the energy of a system our equation becomes:

$$E_o + W = E_f$$
Work

Work can only be done to a system by an external force; a force from something that is not a part of the system.

So if our system is a plane on an aircraft carrier and we come along and push the plane, we can increase the energy of the plane…

We are essentially doing work on the plane.
Work

The amount of work done, and therefore the amount of energy increase that the system will experience is given by the equation:

\[ W = Fd_{\text{parallel}} \]

Meaning, work is the product of the force applied which moves the object a parallel displacement.

There are some important points to understand about this equation.
Work

If the object that is experiencing the force does not move (if $d_{\text{parallel}} = 0$) then no work is done.

The energy of the system is unchanged; a state of equilibrium.
Positive Work

If the object moves in the same direction as the direction of the force (for instance if force and displacement are in the same direction) then the work is positive: $W > 0$.

The energy of the system is increased.

Acceleration occurs due to the unbalanced force. Work is the ability to cause change.
If the object moves in the direction opposite the direction of the force (for instance if force and displacement are in opposite directions) then the work is negative: $W < 0$.

The energy of the system is reduced.

Acceleration occurs due to the unbalanced force. Work is the ability to cause change.
Zero Work

If the object moves in the direction perpendicular the direction of the force (for instance if force and displacement are at right angles) then the work is negative: \( W = 0 \).

The energy of the system is unchanged.

No acceleration occurs due to the fact that no component of force acts in the direction of displacement. In this case, no work is done by the normal force and/or the force of gravity.
Units of Work and Energy

\[ W = Fd_{\text{parallel}} \]

This equation gives us the units of work. Since force is measured in Newtons (N) and displacement is measured in meters (m) the unit of work is the Newton-meter (N-m). And since \( N = \text{kg-m/s}^2 \); a N-m also equals a \( \text{kg-m}^2/\text{s}^2 \).

However, in honor of James Joule, who made critical contributions in developing the idea of energy, the unit of energy is also known as a Joule (J).

\[ \text{J} = \text{N-m} = \text{kg-m}^2/\text{s}^2 \]

\text{Joule} \quad \text{Newton-meter} \quad \text{kilogram-meter}^2/\text{second}^2
Units of Work and Energy

\[ E_0 + W = E_f \]

Since the work changed the energy of a system: the units of energy must be the same as the units of work.

The units of both work and energy are the Joule.

James Joule
1 A +24 N force is applied to an object that moves 10 m in the same direction during the time that the force is applied. How much work is done to the object?
2 A +24 N force is applied to an object that moves 10 m in the opposite direction during the time that the force is applied. How much work is done to the object?

Answer

https://www.njctl.org/video/?v=6XIm1Mwp4g
3. A +24 N force is applied to an object that is stationary during the time that the force is applied. How much work is done to the object?
4 How much force must be applied to an object such that it gains 100J of energy over a distance of 20 m?
5 Over what distance must a 400 N force be applied to an object such that it gains 1600J of energy?

https://www.njctl.org/video/?v=3uNVUJ3IzK
6 A boy rides a bike at a constant speed 3 m/s by applying a force of 100 N. How much work will be done during 100 seconds?

https://www.njctl.org/video/?v=tYBse2UZgrU
7 A horse pulls a sleigh at a constant speed 1.2 m/s by applying a force of 350 N. How much work will be done during 100 seconds?
8 A book is held at a height of 2.0 m for 20 s. How much work is done on the book?

https://www.njctl.org/video/?v=1UId0dPj5pY
9 A barbell of mass "m" is lifted vertically upwards, at a constant velocity, to a distance "h" by an outside force. How much work does that outside force do on the barbell?

A  \( mg \)  
B  \(-mgh\)  
C  \( mgh \)  
D  0  
E  \(-mg\)

Hint: Do a free body diagram to determine a formula for the outside force \( F_{app} \); then use the formula for work: \( W = F_d \parallel d \).  

https://www.njctl.org/video/?v=F-9K5YlPoZk
Forces and Potential Energy
A barbell of mass "m" is lifted vertically upwards a distance "h" by an outside force. How much work does that outside force do on the barbell?

\[ W = Fd_{\parallel} \]

Since \( a = 0 \), \( F_{\text{app}} = mg \)

\[ W = (mg) d_{\parallel} \]

Since \( F \) and \( d \) are in the same direction ... and \( d_{\parallel} = h \)

\[ W = (mg) h \]

\[ W = mgh \]
Gravitational Potential Energy

But we know that in general, 
\[ E_o + W = E_f. \]

If our barbell had no energy to begin with \((E_o = 0)\), then \(W = E_f\)

But we just showed that we did \(W = mgh\) to lift the barbell... so \(mgh = E_f\)

The energy of a mass is increased by an amount \(mgh\) when it is raised by a height "h".
Gravitational Potential Energy

The name for this form of energy is Gravitational Potential Energy (GPE).

\[ \text{GPE} = mgh \]

One important thing to note is that while changes in gravitational potential energy are important, their absolute value is not.
Gravitational Potential Energy

You can define any height to be the zero for height...and therefore the zero for GPE.

But whichever height you choose to call zero, changes in heights will result in changes of GPE. For example, the floor level can be considered zero energy or the ladder level can be zero.
10 What is the change of GPE for a 5.0 kg object which is raised from the floor to a final height of 2.0m above the floor?

https://www.njctl.org/video/?v=neEDh2sD-Pzs
11 As an object falls downward, its GPE always _____.

A  increases  
B  decreases  
C  stays the same
12 What is the change of GPE for a 8.0 kg object which is lowered from an initial height of 2.0 m above the floor to a final height of 1.5m above the floor?
13 What is the change in GPE for a 10.0 kg object which is raised from an initial height of 1.0 m above the floor to a final height of 10.0 m above the floor?
What is the change in height of a 2.0 kg object which gained 16 J of GPE?
15 What is the change in height of a 1/2 kg object which lost 20 J of GPE?

https://www.njctl.org/video/?v=m2Grf6ZpXcU
Kinetic Energy

Imagine an object of mass "m" at rest at a height "h". If dropped, how fast will it be traveling just before striking the ground?

Use your kinematics equations to get a formula for $v^2$.

\[ v^2 = v_o^2 + 2a\Delta x \]

Since $v_o = 0$, $\Delta x = h$, and $a = g$

\[ v^2 = 2gh \]

We can solve this for "gh"

\[ gh = \frac{v^2}{2} \]

We're going to use this result later.
Kinetic Energy

In this example, we dropped an object. While it was falling, its energy was constant...but changing forms.

It only had gravitational potential energy, GPE, at beginning, because it had height but no velocity.

Just before striking the ground (or in the example on the right, before hitting the hand) it only had kinetic energy, KE, as it had velocity but no height.

In between, it had some of both.
Kinetic Energy

Now let's look at this from an energy perspective.

No external force acted on the system so its energy is constant. Its original energy was in the form of GPE, which is "mgh".

\[ W = 0 \text{ and } E_o = mgh \]

Solving for gh yields

\[ gh = \frac{E_f}{m} \]

Now let's use our result from kinematics

\[ v^2/2 = \frac{E_f}{m} \]

Divide both sides by m

\[ E_f = \frac{1}{2}mv^2 \]

This is the energy an object has by virtue of its motion: its kinetic energy.
Kinetic Energy

The energy an object has by virtue of its motion is called its kinetic energy. The symbol we will be using for kinetic energy is KE.

Like all forms of energy, it is measured in Joules (J).

The amount of KE an object has is given by:

\[ KE = \frac{1}{2}mv^2 \]
16 As an object falls, its KE always _____.

A decreases
B increases
C stays the same.
17 A ball falls from the top of a building to the ground below. How does the kinetic energy (KE) compare to the potential energy (PE) at the top of the building?

A  KE = PE
B  KE > PE
C  KE < PE
D  It is impossible to tell.

https://www.njctl.org/video/?v=v_e84J2poGA
18 What is the kinetic energy of a 12 kg object with a velocity of 10 m/s?
19 What is the kinetic energy of a 20 kg object with a velocity of 5 m/s?

https://www.njctl.org/video/?v=UCpXbehZqNE
20. What is the mass of an object which has 2400 J of KE when traveling at 6.0 m/s?
21 What is the mass of an object which has 2000 J of KE when traveling at 10 m/s?

https://www.njctl.org/video/?v=7SyuJO1aU4I
22 A 3 kg object has 45 J of kinetic energy. What is its velocity?
23 A 10 kg object has 100 J of kinetic energy. What is its velocity?

https://www.njctl.org/video/?v=XgU_jLP3LEU
24 If the speed of a car is doubled, the KE of the car is:

A quadrupled
B quartered
C halved
D doubled

Answer

https://www.njctl.org/video/?v=QuLGUEmaN7c
25 If the speed of a car is halved, the KE of the car is:

A quadrupled
B quartered
C halved
D doubled
26 Which graph best represents the relationship between the KE and the velocity of an object accelerating in a straight line?

A

\[
\begin{array}{c}
\text{KE} \\
\hline \\
\text{v}
\end{array}
\]

B

\[
\begin{array}{c}
\text{KE} \\
\hline \\
\text{v}
\end{array}
\]

C

\[
\begin{array}{c}
\text{KE} \\
\hline \\
\text{v}
\end{array}
\]

D

\[
\begin{array}{c}
\text{KE} \\
\hline \\
\text{v}
\end{array}
\]

https://www.njctl.org/video/?v=c9d-rhrSpCY
27 The data table below lists mass and speed for 4 objects. Which 2 have the same KE?

A  A and D
B  B and D
C  A and C
D  B and C

<table>
<thead>
<tr>
<th>Objects</th>
<th>Mass (kg)</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>B</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>4.0</td>
</tr>
<tr>
<td>D</td>
<td>4.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Elastic Potential Energy

Energy can be stored in a spring, this energy is called Elastic Potential Energy.

Robert Hooke first observed the relationship between the force necessary to compress a spring and how much the spring was compressed.
Elastic Potential Energy

It was common for scientists to establish riddles to prove ownership of new ideas in order prevent others for taking credit of new models.

Robert Hooke first reported his findings of how springs function in anagram form.

ceiiinossstttuv

Can you unscramble this? see the next page for the answer.
Elastic Potential Energy

ceiiinossstttuv
Can you unscramble this?

The answer.
ut tensio, sic vis

Latin;
as the tension, so the force
Hooke's Law

\[ F_{\text{spring}} = -kx \]

k represents the spring constant and is measured in N/m.

x represents how much the spring is compressed and is measured as you would expect, in meters.

The - sign tells us that this is a restorative force. (if you let the spring go once it is compressed, it will go back to its original position)
Hooke's Law

\[ F_{\text{spring}} = -kx \]

If we graph the relationship between force and elongation, the mathematical relationship can be experimentally confirmed.
Hooke's Law

\[ F_{\text{spring}} = -kx \]

Varying the displacement/elongation (x)

- Small elongations require small forces
- Large elongations require large forces
Hooke's Law

\[ F_{\text{spring}} = -kx \]

Varying the spring constant \( k \) (the stiffness of the spring)
The spring constant is related to the slope of the line.

Spring Constant = slope of line (Newtons/meter)
Hooke's Law

\[ F_{\text{spring}} = -kx \]

Varying the spring constant \( k \) (the stiffness of the spring)
The spring constant is related to the slope the line.

\[ F \quad (N) \]

\[ x \quad (m) \]
28 Which spring requires a greater force to stretch?

A blue
B green
C the same force is required

https://www.njctl.org/video/?v=V7FjkmolISZk
29  An ideal spring has a spring constant of 25N/m. Determine the force required to elongate/displace the spring by 2 meters.
30 An ideal spring is requires 30 Newtons of force in order to stretch 5 meters. Determine the spring constant (k).
31 A force of 100 newtons is applied to a spring with a constant of 25 N/m. Determine the resulting displacement/elongation.
Elastic Potential Energy

The work needed to compress a spring is equal to the area under its force vs. distance curve.

\[ W = \frac{1}{2} k x^2 \]

Area of a triangle = 1/2 b h

\[ W = \frac{1}{2} (x)(F) \]
\[ W = \frac{1}{2} (x)(kx) \]
\[ W = \frac{1}{2}kx^2 \]

Work = EPE
Elastic Potential Energy

The energy imparted to the spring by this work must be stored in the Elastic Potential Energy (EPE) of the spring:

$$EPE = \frac{1}{2} k x^2$$

*Like all forms of energy, it is measured in Joules (J).*
Elastic Potential Energy

Work done when varying the displacement/enlongation (x)
Remember- the elastic potential energy stored is equal to the area under the curve.

\[ F = kx \]

\[ EPE = \frac{1}{2} kx^2 \]

Remember- large elongations do large amounts of work.
Elastic Potential Energy

Work done when varying the displacement/enlongation 
(x) Remember- the elastic potential energy stored is equal to 
the area under the curve.

\[ F = kx \] (N)

\[ x \] (m)

\[ F = kx \] (N)

\[ x \] (m)

EPE = \( \frac{1}{2} k x^2 \)

EPE is directly proportional to the square of the elongation. Stretching 
the spring twice as far requires twice the force but FOUR times WORK.
Resistance Bands and EPE

Resistance bands are used for resistance training! These bands allow us to get a 'workout' them because stretching the bands requires AND expends energy.

Resistance bands are available in different tensions (spring constants) and are color coded accordingly.
Elastic Potential Energy

Work done when varying the spring constant (k).

$$EPE = \frac{1}{2} k x^2$$

EPE is directly proportional to the value for the spring constant! Similar displacements require different amounts of work. The large spring constant requires more work and stores more elastic potential energy with similar elongation.
32 Determine the elastic potential energy stored in a spring whose spring constant is 250 N/m and which is compressed 8 cm.
33 Determine the elastic potential energy stored in a spring whose spring constant is 500 N/m and which is compressed 24 cm.

Answer: [Video Link]
34 What is the spring constant of a spring that is compressed 5 cm and has 0.65 J of elastic potential energy stored in it?
35 What is the spring constant of a spring that is compressed 10 cm and has 0.65 J of elastic potential energy stored in it?
36 How much does a spring with a spring constant of 500 N/m need to be compressed in order to store 1.75 J of elastic potential energy?
37 How much does a spring with a spring constant of 500 N/m need to be compressed in order to store 7.0 J of elastic potential energy?

https://www.njctl.org/video/?v=nrgGbQ-UCPwk
38 A 3 kg mass compresses a spring 2.5 cm. What is the spring constant?
39 The same 3 kg mass compresses the same spring 2.5 cm. How much elastic potential energy is stored in the spring?

\[ k = 1176 \text{ N/m} \]
40 The same 3 kg mass compresses the same spring 5 cm. How much elastic potential energy is stored in the spring?

Answer

https://www.njctl.org/video/?v=C1OlSzd3kQg
Conservation of Energy

https://www.njctl.org/video/?v=GuIp7Ja-J8E
A roller coaster is at the top of a track that is 80 m high. How fast will it be going at the bottom of the hill?

\[ E_0 + W = E_f \]

\[ E_0 = E_f, \quad W = 0 \]

GPE = KE

\[ mgh = \frac{1}{2}mv^2 \]

\[ v^2 = 2gh \]

\[ v^2 = 2 \times (9.8 \text{ m/s}^2) \times 80 \text{ m} \]

\[ v = 39.6 \text{ m/s} \]
41 A spring gun with a spring constant of 250 N/m is compressed 5 cm. How fast will a 0.025 kg dart move when it leaves the gun?
42 A spring gun with a spring constant of 250 N/m is compressed 15 cm. How fast will a 0.025 kg dart go when it leaves the gun?
A student uses a spring (with a spring constant of 180 N/m) to launch a marble vertically into the air. The mass of the marble is 0.004 kg and the spring is compressed 0.03m.

What is the maximum height the marble will reach?
A student uses a spring (with a spring constant of 360 N/m) to launch a marble vertically into the air. The mass of the marble is .05 kg and the spring is compressed 0.1 m. What is the maximum height the marble will reach?

https://www.njctl.org/video/?v=1uBXRGEL8eA
45 A student uses a spring gun (with a spring constant of 120 N/m) to launch a marble vertically into the air. The mass of the marble is 0.002 kg and the spring is compressed 0.04 m.

How fast will the marble be traveling when it leaves the gun?
46 A roller coaster has a velocity of 25 m/s at the bottom of the first hill. How high was the hill?

https://www.njctl.org/video/?v=rN8dHTR4eL0
47 A roller coaster has a velocity of 50 m/s at the bottom of the first hill. How high was the hill?
48 A 5 kg rock is dropped a distance of 1m onto the spring. The rock compresses the spring 2 cm. What is the spring constant?

Answer

\[ k = 245000 \text{ N/m} \]
49 A 20 kg rock is dropped a distance of 1m onto the spring. The rock compresses the spring 2 cm. What is the spring constant?

Answer

$k = 980,000 \text{N/m}$
A student uses the lab apparatus shown above. A 5 kg block compresses a spring 6 cm. The spring constant is 300 N/m.

What is the block's velocity be when the spring loses all of the stored elastic potential energy?
A student uses the lab apparatus shown above. A 5 kg block compresses a spring 6 cm. The spring constant is 1200 N/m.

What is the blocks velocity be when the spring loses all of the stored elastic potential energy?

https://www.njctl.org/video/?v=UyjbCuJ46fY
52 How much work is done in stopping a 5 kg bowling ball rolling with a velocity of 10 m/s?

Answer

https://www.njctl.org/video/?v=jYIHVvuUs2M
53 How much work is done in stopping a 5 kg bowling ball rolling with a velocity of 20 m/s?

https://www.njctl.org/video/?v=Ztw_l4pDlHg
54 How much work is done in compressing a spring with a 450 N/m spring constant a distance of 2 cm?
55 How much work is done in compressing a spring with a 900 N/m spring constant 11 cm?

https://www.njctl.org/video/?v=2hFBrWDQb4
Power

It is often important to know not only if there is enough energy available to perform a task but also how much time will be required.

Power is defined as the rate that work is done (or energy is transformed) :

\[ P = \frac{W}{t} \]

100 Watt light bulbs convert 100 Joules of electrical energy to heat and light every second.
Since work is measured in Joules (J) and time is measured in seconds (s) the unit of power is Joules per second (J/s).

However, in honor of James Watt, who made critical contributions in developing efficient steam engines, the unit of power is also known as a Watt (W).
Since \( W = Fd_{parallel} \)

Regrouping this becomes

Since \( v = \frac{d}{t} \)

So power can be defined as the product of the force applied and the velocity of the object parallel to that force.
A third useful expression for power can be derived from our original statement of the conservation of energy principle.

\[ P = \frac{W}{t} \]

Since \( W = E_f - E_0 \)

\[ P = \frac{(E_f - E_0)}{t} \]

So the power absorbed by a system can be thought of as the rate at which the energy in the system is changing.
56 A steam engine does 50 J of work in 12 s. What is the power supplied by the engine?

Answer

https://www.njctl.org/video/?v=NXLuYIIRc9F4
57 How long must a 350 W engine run in order to produce 720 kJ of work?

https://www.njctl.org/video/?v=tR-YweBGdI
58 How long must a 350 W engine run in order to produce 360 kJ of work?

Answer

https://www.njctl.org/video/?v=kTTY7D2lnWM
59 A 12 kW motor runs a vehicle at a speed of 8 m/s. What is the force supplied by the engine?

https://www.njctl.org/video/?v=2O7vv19xmo
60  A 24 kW motor runs a vehicle at a speed of 8 m/s. What is the force supplied by the engine?
An athlete pulls a sled with a force of 200N burning 600 Joules of food/caloric energy every second. What is the velocity of the athlete?
62 An athlete pulls a sled with a force of 100N producing 200 Joules of thermal energy due to friction every second. What is the velocity of the athlete?

https://www.njctl.org/video/?v=FVtIpXNesTA
A 3.0 kg block is initially at rest on a frictionless, horizontal surface. The block is moved 8.0m in 2.0s by the application of a 12 N horizontal force, as shown in the diagram below. What is the power developed when moving the block?

A 24  
B 32  
C 48  
D 96