34.0 - Introduction

Radio and television signals, x-rays, microwaves: Each is a form of electromagnetic radiation. If steam and internal combustion engines symbolize the Industrial Revolution, and microprocessors and memory chips now power the Information Revolution, it almost seems that we have neglected to recognize the “Electromagnetic Revolution.” Think about it: Can you imagine life without television sets or cell phones? You may long for such a life, or wonder how people ever survived without these devices!

These examples are from the world of engineered electromagnetic radiation. Even if you think we might all prosper without such technologies to entertain us, do our cooking, carry our messages, and diagnose our illnesses, you would be hard-pressed to survive without light. This form of electromagnetic radiation brings the Sun’s energy to the Earth, warming the planet and supplying energy to plants, and in turn to creatures like us that depend on them. There are primitive forms of life that do not depend on the Sun’s energy, but without light there would be no seeing, no room with a view, no sunsets, and no Rembrandts.

Some of the electromagnetic radiation that reaches your eyes was created mere nanoseconds earlier, like the light from a lamp. Other electromagnetic radiation is still propagating at its original speed through the cosmos, ten billion years or more after its birth. An example of this is the microwave background radiation, a pervasive remnant of the creation of the universe that is widely studied by astrophysicists.

Back here on Earth, this chapter covers the fundamental physical theory of electromagnetic radiation. Much of it builds on other topics, particularly the studies of waves, electric fields and magnetic fields.

34.1 - The electromagnetic spectrum

Electromagnetic radiation ordered by frequency or wavelength.

Electromagnetic radiation is a traveling wave that consists of electric and magnetic fields. Before delving into the details of such waves, we will discuss the electromagnetic spectrum, a system by which the types of electromagnetic radiation are classified.

The illustration of the electromagnetic spectrum above orders electromagnetic waves by frequency and by wavelength. In the diagram, frequency increases and wavelength decreases as you move from the left to the right. The chart’s scale is based on powers of 10. Wavelengths range from more than 100 meters for AM radio signals to as small as 10⁻¹⁶ meters for gamma rays.

All electromagnetic waves travel at the same speed in a vacuum. This speed is designated by the letter c and is called the speed of light. (The letter c comes from celeritas, the Latin word for speed. It might be more accurate to refer to it as the speed of electromagnetic radiation.) The speed of light in a vacuum is exactly 299,792,458 m/s, and it is only slightly less in air.

The unvarying nature of this speed has an important implication: The wavelength of electromagnetic radiation is inversely proportional to its frequency. As you may recall, the speed of a wave equals the product of its frequency and wavelength. This means that if you know the wavelength of the wave, you can determine its frequency (and vice versa). For instance, an electromagnetic wave with a wavelength of 300 meters, in the middle of the AM radio band, has a frequency of 1 × 10⁶ Hz. This equals 3 × 10⁸ m/s, the speed of light, divided by 300 m. The frequencies of electromagnetic waves range from less than one megahertz, or 10⁶ Hz, for long radio waves to over 10²⁴ Hz for gamma rays.

We will now review some of the bands of electromagnetic radiation and their manifestations. The lowest frequencies are often utilized for radio
signals. AM and FM radio waves are typically produced by transmitters that incorporate electric oscillator circuits attached to antennas. The AM and FM radio bands are shown in Concept 1. Microwave radiation is at the upper end of the radio band, and is used for cellular telephone transmissions as well as for heating food in microwave ovens.

Infrared radiation has a higher frequency than microwaves and is associated with heat. It is generated by the thermal vibration or rotation of atoms and molecules.

Visible light is next as we go up the frequency range. It is of paramount importance to human beings, although it occupies only a small portion of the electromagnetic spectrum. Like some other forms of electromagnetic radiation, it is created when atoms emit radiation as their electrons drop from higher to lower energy levels. Light consists of electromagnetic waves that oscillate more than 100 trillion (10^{14}) times a second. The wavelengths of the various colors of light are in the hundreds of nanometers. Red light has the lowest frequency and longest wavelength, while violet light has the highest frequency and shortest wavelength. The visible light spectrum is shown in Concept 2.

The Sun emits a broad spectrum of electromagnetic radiation, including ultraviolet (UV) waves, with frequencies higher than those of visible light. These waves are the main cause of sunburn: Sunscreen lotion is designed to prevent them from reaching and harming your skin. This radiation can also harm your eyes, especially if you wear plastic sunglasses that diminish the amount of visible light reaching your eyes, but do not block UV rays. Typically, you squint when your eyes are exposed to strong light, and this helps protect them. If you wear sunglasses that do not stop ultraviolet light, your pupils will dilate, allowing an extra dose of harmful ultraviolet waves to enter your eyes.

The Earth’s atmosphere, specifically the layer that contains a molecular form of oxygen called ozone, absorbs a great deal of the Sun’s ultraviolet radiation, protecting plants and animals from its harmful effects. However, substances once commonly used in refrigerators and aerosols catalyze ozone-destroying chemical reactions in the atmosphere. Fortunately, the use of such substances has been restricted, but a “hole” in the ozone, a region where the amount of ozone has been significantly depleted, has been created above Antarctica. This hole varies in size from year to year, but on average is approximately the size of North America.

X-rays, the next band of electromagnetic frequencies, are even more dangerous than UV, but they are also useful. Doctors can use them to “see” shadowy images of the inside of the human body because they travel more easily through some tissues than others. Scientists also use them to discern the detailed crystalline structure of materials and to deduce the spatial configuration of complex molecules. NASA launched the Chandra X-ray Observatory in 1999 to capture the phenomena organized in the chart above and illustrated to the right are all forms of electromagnetic radiation. In the same way that you think of both tiny puddle ripples and long, slow ocean swells as being water waves, so you should think about the types of electromagnetic radiation. The frequency of the wave does not alter the fundamental laws of physics that govern it.
**Electromagnetic wave:** A wave consisting of electric and magnetic fields oscillating transversely to the direction of propagation.

Physicist James Clerk Maxwell’s brilliant studies pioneered research into the nature of electromagnetic waves. He correctly concluded that oscillating electric and magnetic fields can constitute a self-propagating wave that he called electromagnetic radiation. His law of induction (a changing electric field causes a magnetic field) combined with Faraday’s law (a changing magnetic field causes an electric field) supplies the basis for understanding this kind of wave.

As the diagrams to the right show, the electric and magnetic fields in an electromagnetic wave are perpendicular to each other and to the direction of propagation of the wave. These illustrations also show the amplitudes of the fields varying sinusoidally as functions of position and time. Electromagnetic waves are an example of transverse waves. The fields can propagate outward from a source in all directions at the speed of light; for the sake of visual clarity, we have chosen to show them moving only along the \( x \) axis.

The animated diagram in Concept 2 and the illustrations below are used to emphasize three points. First, the depicted wave moves away from the source. For example, if you push the “transmit” button on a walkie-talkie, a wave is initiated that travels away from the walkie-talkie.

Second, at any fixed location in the path of the wave, both fields change over time. The wave below is drawn at intervals that are fractions \( T/4 \) of the period \( T \). Look at the point \( P \) below, on the light blue vertical plane. The vectors from point \( P \) represent the direction and strength of the electric and magnetic fields at this point. As you can see, the vectors, and the fields they represent, change over time at \( P \). Concept 2 shows them varying continuously with time at the point \( P \).

Third, the diagrams reflect an important fact: The electric and magnetic fields have the same frequency and phase. That is, they reach their peaks and troughs simultaneously.

A wave on a string provides a good starting point for understanding electromagnetic waves. Both electromagnetic radiation and a wave on a string are transverse waves. The strengths of the two fields constituting the radiation can be described using sinusoidal functions, just as we can use a sinusoidal function to calculate the transverse displacement of a particle in a string through which a wave is moving.

There is a crucial difference, though: Electromagnetic radiation consists of electric and magnetic fields, and does not require a medium like a string for its propagation. Electromagnetic waves can travel in a vacuum. If this is troubling to you, you are in good company. It took some brilliant physicists a great deal of hard work to convince the world that light and other electromagnetic waves do not require a medium of transmission.

Furthermore, when electromagnetic waves radiate in all directions from a compact source like an antenna or a lamp, the radiation emitted at a particular instant travels outward on the surface of an expanding sphere, and its strength diminishes with distance from the source. The waves cannot be truly sinusoidal, since the amplitude of a sinusoidal function never diminishes.

In the sections that follow we will analyze plane waves, which propagate through space, say in the positive \( x \) direction, in parallel planar wave fronts rather than expanding spherical ones. They are good approximations to physical waves over small regions that are distant from the source of the waves. Plane waves never diminish in strength; they can be accurately modeled using sinusoidal functions, and we will do so.
34.3 - Proportionality of electric and magnetic fields

Although the electric and magnetic field vectors of an electromagnetic wave point in perpendicular directions, their magnitudes are strictly proportional to each other at all positions and at all times. We graphically display the magnitudes at a particular instant on the same coordinate system in Equation 1. Their proportionality is expressed in the equation to the right, using a constant \( c \) that depends on two other fundamental physical constants.

This proportionality turned out to have important implications in the study of electromagnetic radiation. Why? Because when calculated, the value of \( c \) was very close to the measured speed of light. This crucial discovery accelerated the understanding of the relationship between electromagnetic radiation such as light or radio waves, and electric and magnetic fields.

Proportionality of electric and magnetic field strengths

In an electromagnetic wave,

\[
\frac{E}{B} = c
\]

\( E = \) electric field strength
\( B = \) magnetic field strength
\( c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \text{speed of wave} \)

34.4 - Calculating the speed of light from fundamental constants

One of the major discoveries of 19th century physics was that light is a form of electromagnetic radiation. By applying and extending their knowledge of electric and magnetic fields, physicists were able both to create electromagnetic radiation (initially radio waves) and to predict the speed of the waves.

James Maxwell published his four laws governing electromagnetic phenomena in 1864, and at the same time he predicted the existence of self-propagating electromagnetic waves. His work enabled him to calculate what the speed of these waves would have to be.

The angular frequency of any wave is \( \omega = 2\pi f \), where \( f \) is its frequency in cycles per second. The angular wave number is \( k = \frac{2\pi}{\lambda} \), where \( \lambda \) is the wavelength. The speed of any wave is \( v = \frac{\omega}{k} \). This means that the speed in terms of the angular frequency and wave number is \( v = (\frac{2\pi}{k})(\frac{\omega}{2\pi}), \) or \( \omega/k \).

Maxwell had already shown that the wave speed \( \omega/k \) is a constant for electromagnetic waves, a constant he had designated as \( c \) and expressed in terms of \( \mu_0 \) and \( \varepsilon_0 \). We state the relationship of \( c \) to these several variables and constants in Equation 1.

The fundamental constants \( \mu_0 \) and \( \varepsilon_0 \) are used elsewhere in physics. For example, the permeability constant is used in equations that describe the magnetic field created by various electric current configurations. The permittivity constant is used to express one form of Coulomb’s law. In other words, these constants determine the strengths of the electric and magnetic forces in the physical universe.

The example problem on the right asks you to repeat Maxwell’s calculation of the value of \( c \). The result is \( 2.998 \times 10^8 \) m/s. In the year 1864, the speed of visible light in a vacuum had been known with fair accuracy for well over a century. The English physicist James Bradley estimated it in 1728 to be \( 3.1 \times 10^8 \) m/s, based on his study of “stellar aberration,” or the apparent change in the positions of stars as the Earth moves around the Sun. Because the calculated and observed speeds were so close, Maxwell’s results provided the first evidence that light is a kind of electromagnetic wave.

Understanding that light is an electromagnetic wave, and knowing the general relationship between frequency and wavelength, sparked the discovery of additional types of electromagnetic radiation. In 1888 Heinrich Hertz created what we would now call radio receivers and transmitters, one of which you see in the illustration above. He proved the existence, and wave-like nature, of radiation having frequencies around 100 MHz.
Speed of an electromagnetic wave

\[ c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]

- \( c \) = speed of electromagnetic wave
- \( \omega \) = angular frequency of wave
- \( k \) = angular wave number of wave
- \( \mu_0 \) = permeability of free space
- \( \varepsilon_0 \) = permittivity of free space

Constant \( \mu_0 = 4\pi \times 10^{-7} \) T·m/A
Constant \( \varepsilon_0 = 8.854 \times 10^{-12} \) C²/N·m²

What is the speed of an electromagnetic wave in a vacuum?

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]

\[ \mu_0 \varepsilon_0 = (4\pi \times 10^{-7})(8.854 \times 10^{-12}) \]

\[ \mu_0 \varepsilon_0 = 1.113 \times 10^{-17} \text{ s}^2/\text{m}^2 \]

\[ \sqrt{\mu_0 \varepsilon_0} = 3.336 \times 10^{-9} \text{ m/s} \]

\[ c = 2.998 \times 10^8 \text{ m/s} \]

34.5 - Creating electromagnetic waves: antennas

Radio antennas create electromagnetic waves. A radio antenna is part of an overall system called a radio transmitter that converts the information contained in sound waves into electromagnetic waves. A radio receiver then reverses the process, converting the signals from electromagnetic waves back to sound waves.

The system depicted to the right shows the fundamentals of a radio transmitter. In the illustrations, the terminals of an AC generator are connected to two rods of conducting material: an antenna. The AC generator produces an emf \( \bar{E} \) that varies sinusoidally over time. The emf drives positive
and negative charges to opposite ends of the antenna. The separation of the charges on the rods produces an electric field. The AC generator causes the amount and sign of the charge on each rod to vary over time, so that the resulting electric field varies in strength and orientation as well. (The flow of charge - that is, the current - also produces a varying magnetic field close to the antenna, part of what is called the near field, which we do not show here.)

The electric field produced by the antenna at each instant in time propagates outward in all directions at the speed of light. For simplicity’s sake, we only show it traveling in the positive x direction in Concept 2. The electric field changes continuously with time and that change induces a magnetic field. To be precise, it induces a magnetic field proportional to the rate of change of the electric flux with respect to time, as described by Maxwell’s law of induction. In turn, the changing magnetic field regenerates the electric field as the wave travels.

This coupling of changes in the magnetic and electric fields enables the electromagnetic wave to cross vast gulfs of space over immense spans of time. Electromagnetic radiation from distant stars, including light, reaches the Earth after billions of years of travel.

How does an antenna differ from other circuits you may have studied in which current flows or charge is stored? Consider a battery-resistor circuit or a battery-capacitor circuit in equilibrium; the current in the first creates a constant magnetic field, and the stored charge in the second creates a constant electric field. Both fields rapidly diminish as they extend outward in space. The crucial difference with the antenna is that not only does charge accumulate at its ends, but the AC generator continually causes the distribution of charge to change. The electric field varies sinusoidally over time, and a constantly changing electric field is the crucial element required to create continuous, self-propagating electromagnetic radiation.

Electromagnetic waves are generated when charges move at nonconstant velocities, as in an antenna. That is, they are generated by accelerating charges. In an antenna, the acceleration is in a straight line. Charged particles moving in uniform circular motion also emit electromagnetic radiation, called synchrotron radiation, due to their centripetal acceleration.

The AM and FM radio bands are located in different parts of the electromagnetic spectrum, and they are used in different ways to broadcast program content. The difference between them consists in how the information they convey is encoded. In amplitude modulated or AM radio, sound waves are encoded by varying the amplitude of a carrier radio wave around some reference value. Changes in amplitude convey the signal. The frequency of the carrier wave is around 1 MHz for AM radio.

In frequency modulated or FM radio, sound is encoded by slightly varying the frequency of the carrier wave around its base frequency. For FM radio, and television, the carrier wave frequency extends upward from around 100 MHz.

**34.6 - Electromagnetic energy: the Poynting vector**

Electromagnetic waves transmit energy that is crucial to life on Earth. The process of photosynthesis turns the energy of sunlight into chemical energy used by plants, and by the animals that subsist on them - or on each other.

This section introduces the Poynting vector, which begins our discussion of the energy transported by an electromagnetic wave. It will prove a useful tool in developing a formula for the intensity of electromagnetic radiation in the next section. The Poynting vector is represented with the letter $S$.

Equation 1 shows you how to calculate the Poynting vector of a wave in terms of its electric and magnetic fields.

The diagram of Equation 1 shows an electromagnetic wave passing through a surface. The instantaneous rate at which energy is transported...
through the surface by the wave, per unit area, is called the **area power density** of the wave. The area power density is equal to the magnitude $S$ of the Poynting vector.

The surface area through which the instantaneous power density is measured is perpendicular to the direction of the wave’s propagation. When radiation reaches a physical surface obliquely, the cosine of its angle with the area vector can be used to calculate the power conveyed to the surface. This is analogous to the calculation of electric or magnetic flux.

As Equation 1 shows, the Poynting vector equals the cross product of the vectors representing the electric and magnetic fields of the electromagnetic radiation, divided by the permeability constant. Since these fields are always perpendicular to one another, the sine of the angle between them, used to evaluate the magnitude of the cross product, always equals one, and can be effectively ignored when calculating the instantaneous area power density $S$. The units of the Poynting vector are watts per square meter.

The direction of $S$ is determined by the right-hand rule. If you apply the rule, wrapping your fingers from $E$ to $B$ and noting the direction of your thumb, you can correctly determine that it is parallel to the direction of propagation of the wave. When $E$ reverses its direction, so does $B$, and the direction of $S$ remains the same, “pointing” (heh, heh) in the direction of the wave’s motion.

As an electromagnetic wave passes through a surface, the strengths of its electric and magnetic fields there change sinusoidally with time. Since the Poynting vector is the product of these fields, it changes sinusoidally over time, as well. In fact, it varies with values between zero and $E_{\text{max}}B_{\text{max}}/\mu_0$, with a frequency twice that of the fields.

If you are curious why it has this frequency, recall from the field equations that $E$ and $B$ are both cosine functions of time at a fixed point. Then use the trigonometric identity $\cos^2 t = (1 + \cos 2t)/2$.
**Intensity: Average rate of power transmission per unit area.**

As you have seen, the instantaneous area power density of an electromagnetic wave equals the magnitude $S$ of the Poynting vector. It is proportional to the product of the strengths of the electric and magnetic fields at any instant in time.

This section focuses on two other properties used to describe the power or energy of a wave: the intensity and the average volume energy density, which are time-average values rather than instantaneous ones.

Why are average values useful? One reason is that the instantaneous area power density of an electromagnetic wave fluctuates very rapidly. For example, for green light it oscillates at $10^{16}$ Hz. Most radiation detectors, including the human eye, cannot distinguish between such a rapid oscillation and a constant power density. The time average of the area power density is called the **intensity** of a wave. Intensity is, roughly speaking, what you perceive as the “brightness” of a wave of visible light. It is analogous to the intensity of a sound wave, and its units are the same: W/m².

As already stated, the magnitude of the Poynting vector describes instantaneous power density and it is a good starting point for determining intensity: This magnitude equals \( E_B/\mu_0 c \). Using the relationship \( E/B = c \), we can restate the instantaneous power density as \( E^2/\mu_0 c \), where the electric field strength \( E \) varies as a sinusoidal function of time.

The average value of any squared sinusoidal function is one half the squared amplitude of the function. Here, this means \( (E^2)_{\text{avg}} = \frac{E_{\text{max}}^2}{2} \). Substituting this time-average value into the equation \( S = \frac{E^2}{\mu_0 c} \) gives the formula for intensity, symbolized by the letter \( I \), that appears in Equation 1.

With oscillating quantities, scientists often prefer to express an average value in terms of the root mean square. For a sinusoidal function, this equals the maximum value of the function divided by the square root of two. We express intensity in terms of the root mean square of the electric field with the second boxed formula in Equation 1, which follows from the first boxed equation and the definition of the root mean square.

When scientists measure the “power density” of an electromagnetic wave, they measure the amount of energy transported by the wave per unit time through a unit of surface area. An alternate approach to density is to select a fixed instant in time and measure the amount of electric and magnetic energy contained in a volume of space at that instant. When scientists take this approach, they are talking about **energy density**, or volume energy density. The units of energy density are joules per cubic meter, J/m³.

In Equations 2 and 3 on the right, you see equations for energy density. The energy density \( u_\text{e} \) of an electric field \( \mathbf{E} \) is \( \frac{E^2}{\mu_0 c^2} \), a result derived in the study of capacitors.

The energy density \( u_\text{m} \) of a magnetic field \( \mathbf{B} \) is \( B^2/2\mu_0 \), a relationship derived in the study of inductors. The formulas hold true, not just for the fields in circuit components, but for any electric and magnetic fields. Since these fields are constantly changing in an electromagnetic wave, the equations reflect **instantaneous** energy densities.

It can be shown, using the relationships \( E = B/c \) and \( c^2 = 1/\epsilon_0 \mu_0 \), that even as they vary over time, the electric and magnetic energy densities are equal to each other at all times and all positions in an electromagnetic wave. You see this equality stated in Equation 2. From the standpoint of energy, electromagnetic radiation is “half electric” and “half magnetic.” The **total** energy density \( u \) is defined as the sum of these two equal quantities. This is shown as the final equation in Equation 2.

In a fashion similar to the one used to calculate intensity, a formula for the average over time of the total energy density in terms of \( E_{\text{max}} \) can be derived. It is shown in Equation 3.

The “Z Accelerator” in Albuquerque, New Mexico (pictured above) has achieved the controlled fusion of a BB-sized capsule of deuterium (“heavy” hydrogen whose atoms have a neutron plus a proton in the nucleus) by bombarding it with 20 trillion watts of x-radiation for the brief period of five nanoseconds. The average total energy density of this bombardment is \( 3 \times 10^{12} \) J/m³. The hydrogen in the sample fuses to form helium and produces enough energy to power a dim light bulb for one ten-thousandth of a second. Someday, such fusion reactors may provide a practical source of electric power.

The first example problem on the right poses a more mundane challenge. It asks: What are the maximum electric and magnetic field strengths of white light emanating from a computer screen? The intensity of the light is given.
Electric, magnetic energy densities

\[ u_E = \frac{\varepsilon_0 E^2}{2} \quad u_B = \frac{B^2}{2\mu_0} \]

Since \( E^2/B^2 = c^2 = 1/\mu_0\varepsilon_0 \), then

\[ u_E = u_B \]

The total energy density is

\[ u = u_E + u_B = 2u_E = 2u_B \]

\( u_E \) = electric field energy density
\( u_B \) = magnetic field energy density
\( u \) = total energy density

Average energy per unit volume

Average value of \( u \) over time

\[ u_{\text{avg}} = \frac{\varepsilon_0 E_{\text{max}}^2}{2} \]

\( u_{\text{avg}} \) = average total energy density
Units: joules per cubic meter \((J/m^3)\)
What are the maximum field strengths $E_{\text{max}}$ and $B_{\text{max}}$ for the light emanating from the white square above?

$$I = \frac{E_{\text{max}}^2}{2\mu_0 c}, \text{ so } E_{\text{max}} = \sqrt{2\mu_0 c I}$$

$$E_{\text{max}} = \sqrt{2(4\pi \times 10^{-7})(3.0 \times 10^8)}(19)$$

$E_{\text{max}} = 120 \text{ V/m}$

$B_{\text{max}} = E_{\text{max}} / c$

$B_{\text{max}} = (120 \text{ V/m})/(3.0 \times 10^8 \text{ m/s})$

$B_{\text{max}} = 4.0 \times 10^{-7} \text{ T}$

A fusion reactor focuses $2 \times 10^{13}$ watts of x-rays on this deuterium capsule for $5 \times 10^{-9}$ s. What is the average energy density of the radiation? What is the electric field amplitude of the radiation?

$$u_{\text{avg}} = \frac{E}{V_0} = \frac{P \cdot t}{V_0}$$

$$u_{\text{avg}} = \frac{(2 \times 10^{13} \text{ W})(5 \times 10^{-9} \text{ s})}{\frac{1}{2} \pi (0.002)^3 \text{ m}^3}$$

$$u_{\text{avg}} = 3 \times 10^{12} \text{ J/m}^3$$

$$u_{\text{avg}} = \frac{\varepsilon_0 E_{\text{max}}^2}{2}, \text{ so } E_{\text{max}} = \sqrt{\frac{2u_{\text{avg}}}{\varepsilon_0}}$$

$$E_{\text{max}} = \sqrt{\frac{2 \left(3 \times 10^{12} \frac{J}{\text{m}^3}\right)}{8.85 \times 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2}} = 8 \times 10^{11} \text{ V/m}$$
Almost all the energy we use on Earth originates in the Sun and arrives in the form of electromagnetic radiation. Roughly the same amount of solar power reaches the planet throughout the year, yet many places on the globe experience significant seasonal variations in the rate at which they receive this energy.

The cause of seasonal changes in the Earth's climate is the tilt of its axis, the line about which it rotates. The illustration in Concept 1 shows the position of the Earth in its orbit at different times of the year, as well as the direction in which the axis points. The axis is tilted at a 23.5° angle away from a line perpendicular to the Earth's orbital plane. It always points towards the same direction in space (which is why Polaris remains the North Star throughout the year).

March 21 and September 22 are known as the equinoxes. The name refers to the equal lengths of night and day (12 hours each) for all locations on Earth on these dates. December 21 and June 21 are the solstices. As the season progresses from autumn to winter, the Sun rises to a lower high point in the sky each day, and the days get shorter. On the winter solstice (meaning "sun stop"), the Sun stops getting lower and begins to rise to a higher apex each day, as it does through the rest of the winter and spring. The opposite happens after the summer solstice – the Sun once again peaks at a lower point each day. (The dates of the equinoxes and solstices vary from year to year, but are always around the 21st of the month.)

While June 21 is the summer solstice in the Northern Hemisphere, it is the winter solstice for the Southern Hemisphere. Concept 2 illustrates why this is true. It shows a Northern Hemisphere city, Beijing, and a Southern Hemisphere city, Perth, at noon on June 21.

Imagine a solar collection plate of area \(A\) lying flat on the ground, tangent to the Earth's surface in either of these cities. Light rays from the Sun arrive approximately parallel to the plane of the Earth's orbit. On June 21, the sunlight intersects the collecting plate in Beijing at a steeper angle than in Perth. Because Beijing is receiving sunlight more vertically, the energy from that light is more concentrated – Beijing is receiving more power over the area of its collecting plate.

In Perth, a smaller amount of sunlight is being spread over the same collecting area because of the oblique, slanting angle at which it hits the plate. The plate absorbs less power. It is summer in Beijing, and winter in Perth.

You can also look at the situation from a flux perspective. In Beijing the sunlight is closer to being parallel to the area vector of the plate (considered to be pointing into the Earth), meaning there is a greater flux of light through the plate in Beijing. The greater flux means that more solar power is being received there.

Six months later, on December 21, the situation will be reversed: Perth will receive more direct sunlight than Beijing.

Generally speaking, locations farther than Beijing or Perth from the equator experience a greater variation in the power they receive throughout the year, and places closer to the equator experience less change. At the poles – dark six months of the year – this difference is extreme.

Some people mistakenly believe that the seasons are due to the eccentricity of the Earth's orbit – the fact that the Earth's distance from the Sun changes throughout the year. Your first clue that this belief is false is the observation that summer in the Southern Hemisphere occurs at the same time as winter in the Northern Hemisphere (you merely have to make a long-distance phone call to confirm this). In fact, during winter in the Northern Hemisphere, the Earth is actually closer to the Sun than in summer. The reason the eccentricity has only a slight effect is that the Earth's orbit is only slightly elliptical. The annual variation in insolation due to the eccentricity of the Earth's orbit is about 7%, in contrast to an approximately 110% increase from winter to summer (at the latitude of Beijing) due to axial tilt.

In the next section we calculate the change in flux due to axial tilt in Beijing.
Assume that sunlight arrives parallel to the plane of the Earth’s orbit and that its intensity has a constant value throughout the year.

To quantify the amount of sunlight, use the Poynting vector \( \mathbf{S} \), which equals the light’s power per unit area. In the case of electric or magnetic fields, flux has been defined as the magnitude of a field vector times a component of an area vector. Here, \( \mathbf{S} \) fulfills the role of the field vector and the dimensions of solar flux are “power received”: power per unit area, times area, equals power. Beijing is at a latitude of 39.9°, so the angle the collecting plate’s area vector makes with the Poynting vector on June 21 is 39.9° – 23.5° = 16.4°. On December 21 it is 39.9° + 23.5° = 63.4°.

Variables

- instantaneos power per unit area reaching Earth: \( S \)
- area vector of Earth-horizontal collecting plate: \( A \)
- angle between Poynting and area vectors: \( \theta \)
- solar flux on June 21: \( \Phi_{\text{Jun}} \)
- solar flux on December 21: \( \Phi_{\text{Dec}} \)

What is the strategy?

1. Calculate the solar flux on June 21 as a multiple of the magnitudes of \( \mathbf{S} \) and \( A \).
2. Calculate the solar flux on December 21 as a multiple of the magnitudes of \( \mathbf{S} \) and \( A \).
3. Divide to obtain the ratio of solar fluxes.

Physics principles and equations

Equation of flux

\[ \Phi = \mathbf{S} \cdot A = S A \cos \theta \]

Step-by-step solution

Start by calculating the flux on June 21. Use the illustration.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \Phi_{\text{Jun}} = S A \cos \theta )</td>
</tr>
<tr>
<td>2.</td>
<td>( \Phi_{\text{Jun}} = S A \cos 16.4^\circ )</td>
</tr>
<tr>
<td>3.</td>
<td>( \Phi_{\text{Jun}} = 0.96 S A )</td>
</tr>
</tbody>
</table>

Now calculate the flux on December 21.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td>( \Phi_{\text{Dec}} = S A \cos \theta )</td>
</tr>
<tr>
<td>5.</td>
<td>( \Phi_{\text{Dec}} = S A \cos 63.4^\circ )</td>
</tr>
<tr>
<td>6.</td>
<td>( \Phi_{\text{Dec}} = 0.45 S A )</td>
</tr>
</tbody>
</table>

Find the ratio of the flux in June to that in December.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>( \frac{\Phi_{\text{Jun}}}{\Phi_{\text{Dec}}} = \frac{0.96 S A}{0.45 S A} = 2.1 )</td>
</tr>
</tbody>
</table>
The ratio of sunlight power received in June versus December in Beijing is about 2.1:1. This means the increase in flux from winter to summer is 110%. Greater ratios hold for locations farther than Beijing from the equator, while ratios approaching 1.0 hold for places closer to the equator.

As stated before, this change is much greater than the 7% worldwide variation in flux due to the Earth approaching and receding from the Sun in the course of its yearly orbit.

### 34.10 - Radiation intensity and distance

So far, we have modeled electromagnetic waves as plane waves, whose average power is the same everywhere in space.

Now we want to consider spherical waves, waves emanating from a point source and propagating outward in all directions. On the right you see a spherical wave emanating from a point source and passing through a fixed spherical surface of radius $r$. With this kind of wave, the intensity of the radiation decreases as the distance from the source of the radiation increases. We will assume that the energy of the radiation is emitted from the source at a constant rate: The power of the source does not change.

To understand how intensity varies with distance, first consider how a spherical wave spreads. It travels in all directions at the same rate (the speed of light, for an electromagnetic wave). At any instant in time, every point on a wave front is the same distance from the source of the wave. This means the wave’s energy is distributed over a spherical surface, since all the points on a sphere are the same distance from its center.

The propagation of spherical wave fronts is more or less analogous to what happens when a pebble is dropped into a still pond: Wave fronts move out in concentric circles from where the pebble strikes the water. However, spherical electromagnetic wave fronts spread out in three dimensions, not two.

To discuss intensity, say the source emits electromagnetic radiation with power $P$ and consider the radiation passing out through a fixed sphere of radius $r$. The energy emitted at the source during a time interval $\Delta t$ will pass through the whole surface of this sphere during a later time interval of equal duration, so the power is the same at the source as at the surface. Intensity equals average power per unit area. Using the fact that the area of the sphere equals $4\pi r^2$, we can state the intensity equation in Equation 1. The equation shows that the intensity of radiation from a point source obeys an inverse square law: It diminishes as the square of the distance from the source. (A similar equation describes the intensity of spherically expanding sound waves.)

Example 1 asks you to calculate the intensity of sunlight at the distance of the Earth’s orbit. At this distance, the Sun can be considered a point source of electromagnetic radiation. The Sun actually emits radiation of varying intensity, mostly as visible light, over a variety of wavelengths. The result stated is the total intensity for all wavelengths of visible light: roughly speaking, what we perceive as “white” light.

The intensity of sunlight falling on the Earth’s atmosphere is about 70 times the intensity of the white light emerging from your computer screen, which is around 19 W/m². The intensity of sunlight reaching the Earth’s surface is less, due to factors such as the reflection, absorption and scattering of the light in the atmosphere.

### Intensity and distance

$$ I = \frac{P}{4\pi r^2} $$

- $I$ = radiation intensity
- $P$ = power of radiation source
- $r$ = distance from radiation source

### Example 1

The Sun emits $3.91 \times 10^{26}$ watts of radiation. What is its intensity at the distance of the Earth, $1.50 \times 10^{11}$ m?

$$ I = \frac{P}{4\pi r^2} $$

$$ I = \frac{(3.91 \times 10^{26} \text{ W})}{4\pi (1.50 \times 10^{11} \text{ m})^2} $$

$$ I = 1380 \text{ W/m}^2 $$
34.11 - Interactive checkpoint: an illuminated manuscript

The desk lamp in the picture uses a 100 W bulb, producing white light with an efficiency of 6.50%. The part of the book directly underneath, where the intensity of the light is greatest, is 25.0 cm from the bulb. What is the amplitude of the electric field component of the light falling on the indicated portion of the book’s page?

Answer:

\[ E_{\text{max}} = \text{N/C} \]

34.12 - Intensity and field strength around a dipole antenna

Radio wave antennas produce electromagnetic radiation by accelerating charges back and forth along a conductor. The waves, which travel outward from the antenna in many directions, constitute the radiation field of the antenna.

Antennas produce two kinds of fields: In addition to the radiation field, they produce a near field that is fundamentally different from the radiation field, and of less importance since its strength diminishes rapidly with distance from the antenna. In this section we discuss the differences between these two fields and show how the radiation field expands not just along the x axis, but throughout three-dimensional space in a pattern of varying intensities.

First, we discuss the near field, which like the radiation field has electric and magnetic components. Concept 1 shows a dipole antenna at three stages in its operation. In the first stage, the separation of charges on the antenna is at its maximum, and no current is flowing. At this point, the three-dimensional electric field around the antenna closely resembles that of an ideal electric dipole. This means its strength diminishes as the cube of the distance \( r \) from the center of the dipole.

In the second stage of operation, charge is starting to flow from the top to the bottom of the antenna. The electric field is diminishing in strength, and the current is beginning to create a circular magnetic field around the antenna. The magnetic field’s direction is given by the right-hand rule for currents.

In the third stage, the charge separation is zero, so there is no electric field, but the current flow is at a maximum, generating the maximum magnetic field. The current will continue to flow downward in the antenna until it builds up a maximum separation of charge in the opposite direction, when once again the electric field will be at a maximum (directed upward), and the magnetic field will be zero.

By inspecting the diagram, you can see that the electric and magnetic components of the near field oscillate (they vary sinusoidally) and are perpendicular to each other, but they do not together constitute the electric and magnetic components of a self-propagating wave. This is because they are out of phase: The magnetic field lags the electric field by 90°. Since the strength of the near field decreases as \( 1/r^3 \), and it cannot propagate itself, at moderate distances from the antenna this field is barely detectable.

The self-propagating electromagnetic radiation field of the antenna arises from the sinusoidal variation within the electric and magnetic fields discussed above, rather than from their quite limited extent or from their nonexistent interaction with each other. And the radiation field decreases in strength much more gradually. Concept 2 shows a portion of the radiation field emerging from an antenna. Its intensity is measured in the equatorial plane of the dipole at two distances that are large enough that the antenna approximates a point source. As shown in a previous section, the intensity of such radiation decreases with increasing distance from the source: It is proportional to \( 1/r^2 \). For example, at twice the distance from the antenna, the power of the wave is spread out over an area four times as large. Since intensity is proportional to the square of the amplitude of the electric field, the strength of the electric field itself must be decreasing as \( 1/r \).

In our previous discussion of antennas, for simplicity’s sake we showed a single electromagnetic wave propagating along the positive x axis. However, waves actually propagate in many directions from an antenna. The illustration in Concept 3 shows the complex pattern of electric
fields around a dipole antenna at an instant in time. The illustrated electric component of the radiation field consists of an expanding series of concentric tori, or “donuts,” circling the equator of the dipole. The donuts farther away from the antenna were generated by charges on the antenna at earlier times. The field orientations in the donuts alternate due to the alternating polarity of the antenna charge separation. The field lines of the magnetic field component of the radiation field, which are not shown, would consist of concentric rings around the antenna, everywhere perpendicular to the electric field lines, and periodically alternating in orientation.

An ideal source that generates a spherical radiation field is called an isotropic radiator. We used such an ideal radiator when we developed the formula expressing intensity as a function of the distance from a source of radiation. Concept 3 shows that the radiation field generated by a dipole antenna does not propagate in a series of expanding spheres. It is strongest where the field lines are closest together, that is, around the equator of the dipole. It is weakest along the axis of the dipole, where it has zero strength.

The dipole antennas we have been discussing are sometimes called half-wave dipole antennas. This is because they efficiently emit radiation with a wavelength equal to twice the combined length of the two antenna rods. The frequency of the electromagnetic wave is the same as the frequency of the alternating current driving the antenna. When the length of the antenna is half a wavelength, the wave’s frequency is in resonance with the natural oscillatory motion of electric charges in the antenna, resulting in efficient operation.

34.13 - Radiation pressure

Science fiction writers like to ponder space travel. Here is one of their creative ideas: giant space sails that use light instead of wind to move spacecraft. Radiation from the Sun or other stars exerts a force on the spaceship’s sail, propelling the craft through space.

This concept has moved beyond science fiction. The 1964 Echo 2 satellite, a highly reflective low-mass Mylar balloon with a diameter of 30 m, experienced detectable acceleration as a result of sunlight pressure. A group called the Planetary Society is planning to launch a spacecraft that will use radiation pressure as its motive force.

The proposed propulsion of spacecraft depends on the fact that light has linear momentum. This assertion would have come as no surprise to Isaac Newton, who is almost as famous for his use of a prism to decompose white light into its constituent colors as he is for his studies of gravity and motion. He theorized that light consisted of streams of tiny particles that he called “corpuscles,” and was thereby able to explain many of its observed properties.

According to a particle theory of light, a stream of light corpuscles striking a surface imparts momentum to it, much as a hail of bullets might cause a target to move. This effect is shown in Concept 2.

During the 17th century, Newton’s particle theory of light vied with a wave theory championed by his rivals Christiaan Huygens and Robert Hooke. For over a hundred years Newton’s corpuscles held sway, until Thomas Young’s 1801 diffraction experiments (the topic of another chapter in this book) made the wavelike nature of light incontestable.

Maxwell, with his famous treatise on the nature and speed of electromagnetic waves, and his apparent discovery that light itself is a form of electromagnetic radiation, seemed to drive the final nail into the coffin of the particle theory. He was able to explain radiation pressure by showing that electromagnetic waves would possess momentum exactly as particles did. The debate did not end there, however. Albert Einstein used a phenomenon called the photoelectric effect to demonstrate the particle nature of light. Today, light is understood to exhibit both wavelike and particelike properties.

To show how much pressure radiation exerts on an object, we start with an equation developed by Maxwell that is shown in Equation 1. He determined that the amount of momentum transferred by radiation to a “blackbody” equals the amount of energy it transfers to it, divided by the speed of light. A blackbody is an object that absorbs all the radiation that strikes it, and reflects none.

A perfectly reflective surface will have twice as much momentum transferred to it by incident electromagnetic radiation as an illuminated blackbody does, since each wave reverses direction as it reflects off the surface. (This is akin to modeling how gas molecules that collide elastically with a surface collectively exert pressure on it; their change in velocity is [negative] two times their initial velocity, so their change in momentum is [negative] two times their initial momentum.) Equation 2 states that the radiation pressure exerted on a perfect reflector is proportional to the intensity of the radiation falling on it: \( P = 2I/c \). The factor of two in the proportion corresponds to the assumption that the radiation is reflected: For a blackbody the analogous equation would be \( P = I/c \).
Pushing a spaceship is one way that light’s pressure can be applied. Scientists have also shown how laser light can be used to cool a gas. The collisions of light waves with the molecules of the gas are used to reduce the momenta of these particles, which reduces their thermal velocities. A system of six lasers is able to create a cubical region containing “optical molasses” that traps gas molecules with a temperature as low as 2 µK (two millionths of a Kelvin above absolute zero). This work won a Nobel Prize in 1997 for Steven Chu, Claude Cohen-Tannoudji, and William Phillips.

Farther from the Earth, the effects of sunlight pressure on the microscopic particles boiling off a comet as it passes close to the Sun are spectacular: The particles form a “tail” that streams off the comet away from the Sun like a windblown plume of smoke, no matter what direction the comet is moving.

**Derivation** We will use Maxwell’s impulse equation, shown in Equation 1, to derive the equation relating pressure and intensity, shown in Equation 2. We will first derive it for an object that absorbs all the radiation energy falling on it (a blackbody); then, the pressure is doubled for a perfect reflector.

**Variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>pressure of electromagnetic radiation</td>
</tr>
<tr>
<td>$A$</td>
<td>area of object subject to radiation</td>
</tr>
<tr>
<td>$F$</td>
<td>force exerted by radiation</td>
</tr>
<tr>
<td>$p$</td>
<td>momentum of radiation</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time interval</td>
</tr>
<tr>
<td>$U$</td>
<td>total energy of radiation</td>
</tr>
<tr>
<td>$c$</td>
<td>the speed of light</td>
</tr>
<tr>
<td>$I$</td>
<td>intensity of radiation</td>
</tr>
</tbody>
</table>

**Strategy**

1. Express the radiation pressure on the object as force divided by area, and further express the force as the average rate of change of momentum.
2. Use Maxwell’s expression for the impulse of a wave to find the average rate of change of momentum in terms of the energy $\Delta U$ transferred by radiation to the absorbing object. Substitute this expression for the rate of change of momentum into the equation for pressure derived in the previous stage.
3. Finally, use the relationship of $\Delta U$ to intensity to get the radiation pressure in terms of intensity.

**Physics principles and equations**

We will use the definitions of pressure and impulse.

$$P = \frac{F}{A}, \quad \Delta p = \frac{F}{\Delta t}$$

Maxwell’s equation for the momentum transferred to an absorbing object by electromagnetic radiation during the time interval $\Delta t$ is

$$\Delta p = \frac{\Delta U}{c}$$

To simplify the derivation, we assume that the rate of energy transfer is constant. Finally, the intensity of electromagnetic radiation is defined as the power it conveys per unit area, and power is the rate of change of energy.

$$I = \frac{P}{A}, \quad P = \frac{\Delta U}{\Delta t}$$

**Pressure on a perfect reflector**

$$P = \frac{2I}{c}$$

- $P =$ pressure
- $I =$ intensity
- $c =$ speed of light
Step-by-step derivation

We start with the definition of pressure, and use the definition of impulse to replace the force variable appearing in the equation.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( P = \frac{F}{A} )\n</td>
</tr>
<tr>
<td>2.</td>
<td>( F = \frac{\Delta p}{\Delta t} )\n</td>
</tr>
<tr>
<td>3.</td>
<td>( P = \frac{1}{A} \frac{\Delta p}{\Delta t} )\n</td>
</tr>
</tbody>
</table>

Here, we use Maxwell’s equation for the momentum transferred to a blackbody absorber by electromagnetic radiation during a time interval \( \Delta t \). We use this equation to replace the average rate of change of momentum appearing above with an expression that involves energy.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td>( \Delta p = \frac{\Delta U}{c} )       \n</td>
</tr>
<tr>
<td>5.</td>
<td>( \frac{\Delta p}{\Delta t} = \frac{(\Delta U/\Delta t)}{c} ) \n</td>
</tr>
<tr>
<td>6.</td>
<td>( P = \frac{1}{A} \frac{(\Delta U/\Delta t)}{c} ) \n</td>
</tr>
</tbody>
</table>

Intensity provides a simpler, more direct way to calculate pressure. We use the relationship of energy and intensity to relate pressure to intensity.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>( P = \frac{1}{c} \frac{(\Delta U/\Delta t)}{A} ) \n</td>
</tr>
<tr>
<td>8.</td>
<td>( I = \frac{P}{A} = \frac{(\Delta U/\Delta t)}{A} ) \n</td>
</tr>
<tr>
<td>9.</td>
<td>( P = \frac{I}{c} ) \n</td>
</tr>
<tr>
<td>10.</td>
<td>( P = \frac{2I}{c} ) \n</td>
</tr>
</tbody>
</table>

Example 1 asks you to calculate the force exerted by sunlight on a piece of paper. You must first find the light pressure. If the paper were a perfect blackbody, reflecting 0% of the light falling on it, then you would use the formula \( P = I/c \). If it were a perfect reflector, reflecting 100% of the incident light, then you would use \( P = 2I/c \).

Since the paper in fact reflects 80% of the light falling on it (scientists say it has an albedo of 0.8), the appropriate intermediate formula to use is \( P = 1.8I/c \). The downward force of the light on the paper is on the order of \( 10^{-7} \) N, comparable to the weight of the graphite in the penciled “x” on the upper left corner of the sheet.

You may wonder why, in a discussion of radiation pressure, no mention has been made of the angle at which electromagnetic radiation strikes a surface: Wouldn’t oblique radiation exert less pressure than perpendicular radiation? The answer is yes, but this is already accounted for in Equation 2. When radiation striking a surface makes a nonzero angle with its area vector, then the intensity of the illumination is itself reduced, being proportional to the cosine of the angle of incidence.
You have just purchased your brand new Dystis Extranuator space yacht. It carries a vast square sail, 10 kilometers by 10 kilometers, of perfectly reflective material, computer controlled through a harness of InvisiFlex monomolecular cables to have a perpendicular orientation to the incident sunlight at all times. This sleek little puppy, including sails and rigging, has a mass of 9750 kg.

You decide to go for a jaunt heading directly away from the Sun, starting at the distance of the Earth’s orbit. How fast will you be sailing after one hour? And how far will you have traveled? For comparison, how fast will you be going when you cross the orbit of Mars?

The power output of the Sun and the orbital distances are given in the table below.

To simplify the problem, we ignore the effects of Earth’s gravity, and that of all other bodies. Also, without significant error we may use the initial acceleration of the spacecraft as a “constant” acceleration during the first hour’s journey.

**First two questions: speed and distance after one hour**

**Variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>pressure of sunlight on sail</td>
<td>$P$</td>
</tr>
<tr>
<td>intensity of sunlight striking sail</td>
<td>$I$</td>
</tr>
<tr>
<td>the speed of light</td>
<td>$c = 3.00 \times 10^8$ m/s</td>
</tr>
<tr>
<td>power output of the Sun</td>
<td>$P_s = 3.91 \times 10^{26}$ W</td>
</tr>
<tr>
<td>distance from the Sun</td>
<td>$r$</td>
</tr>
<tr>
<td>Earth’s distance from the Sun</td>
<td>$r_i = 1.50 \times 10^{11}$ m</td>
</tr>
<tr>
<td>Mars’ distance from the Sun</td>
<td>$r_f = 2.28 \times 10^{11}$ m</td>
</tr>
<tr>
<td>force exerted by sunlight on sail</td>
<td>$F$</td>
</tr>
<tr>
<td>area of sail</td>
<td>$A = (1.00 \times 10^4 \text{ m})^2$</td>
</tr>
<tr>
<td>“constant” acceleration of spacecraft</td>
<td>$a$</td>
</tr>
<tr>
<td>mass of spacecraft</td>
<td>$m = 9.75 \times 10^3$ kg</td>
</tr>
<tr>
<td>speed of spacecraft</td>
<td>$v$</td>
</tr>
<tr>
<td>initial speed of spacecraft</td>
<td>$v_i = 0$ m/s</td>
</tr>
<tr>
<td>final speed of spacecraft</td>
<td>$v_f$</td>
</tr>
<tr>
<td>elapsed time from start of trip</td>
<td>$t$</td>
</tr>
<tr>
<td>distance traveled from start</td>
<td>$\Delta x$</td>
</tr>
</tbody>
</table>

**What is the strategy?**

1. Use the equations for radiation pressure and intensity to determine the pressure of sunlight on the sail.
2. Use the definition of pressure and Newton’s second law to calculate the acceleration of the spacecraft.
3. Then, use two motion equations to determine the final speed and distance traveled by the ship after one hour.

**Physics principles and equations**

We will need to use the equation for the pressure of electromagnetic radiation on an ideal reflector.

$$P = \frac{2I}{c}$$

The intensity of solar radiation depends on the distance $r$ from the Sun.

$$I = \frac{P_s}{4\pi r^2}$$
Pressure is force per unit area.

\[ P = \frac{F}{A} \]

Newton’s second law.

\[ F = ma \]

We will need two motion equations from the study of kinematics, which we may use because we are assuming that the acceleration of the spacecraft is constant.

\[ v_f = v_i + at \]

\[ \Delta x = v_i t + \frac{1}{2} at^2 \]

**Step-by-step solution**

First, we find the pressure of sunlight on the sail in terms of the power of the Sun and the distance of the spacecraft from it.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( P = \frac{2I}{c} )</td>
</tr>
<tr>
<td>2.</td>
<td>( I = \frac{P_s}{4\pi r^2} )</td>
</tr>
<tr>
<td>3.</td>
<td>( P = \frac{P_s}{2\pi r^2} )</td>
</tr>
</tbody>
</table>

We use the definition of pressure, and Newton’s second law, to find the acceleration of the spacecraft.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td>( F = AP )</td>
</tr>
<tr>
<td>5.</td>
<td>( F = \frac{AP_s}{2\pi r^2} )</td>
</tr>
<tr>
<td>6.</td>
<td>( a = \frac{F}{m} )</td>
</tr>
<tr>
<td>7.</td>
<td>( a = \frac{AP_s}{2\pi cmr^2} )</td>
</tr>
</tbody>
</table>

Now, we calculate the acceleration using the values supplied in the table above. Then we use motion equations to calculate the ship’s speed and distance traveled after one hour.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td>( \alpha = \frac{(1.00 \times 10^{-4} m)^2 \times (3.91 \times 10^{-25} W)}{2\pi (3.00 \times 10^8 m/s^2)} (9.75 \times 10^2 kg) (1.50 \times 10^{-11} m)^2 )</td>
</tr>
<tr>
<td></td>
<td>( \alpha = 9.46 \times 10^{-2} m/s^2 )</td>
</tr>
<tr>
<td>9.</td>
<td>( v_f = v_i + at )</td>
</tr>
<tr>
<td>10.</td>
<td>( v_i = 0 + (9.46 \times 10^{-2} m/s^2)(3.60 \times 10^3 s) )</td>
</tr>
<tr>
<td></td>
<td>( v_f = 341 \text{ m/s} )</td>
</tr>
<tr>
<td>11.</td>
<td>( \Delta x = v_i t + \frac{1}{2} at^2 )</td>
</tr>
<tr>
<td>12.</td>
<td>( \Delta x = \frac{1}{2} (9.46 \times 10^{-2} m/s^2)(3.60 \times 10^3 s)^2 )</td>
</tr>
<tr>
<td></td>
<td>( \Delta x = 6.13 \times 10^5 \text{ m} )</td>
</tr>
</tbody>
</table>

Your ship is moving at a good clip at the end of the first hour, about as fast as a jet plane in the Earth’s atmosphere, and it has gone a fair distance: 613 km.

**Third question: speed at the orbit of Mars.** The third question posed above, how fast are you going when your yacht reaches the orbit of
Mars, requires the use of a different motion equation, since the acceleration and the distance traveled are known but the elapsed time is not.

Variables

In the previous steps we calculated the acceleration of the ship at the Earth’s orbit, and accurately used this value as its “constant” acceleration for the first hour’s journey. Since your acceleration decreases significantly at greater distances from the Sun, but we still wish to use a motion equation that requires constant acceleration, we will use a mean acceleration as the “constant” acceleration for the longer journey. We already calculated this value for you at a point halfway between the orbits of the Earth and Mars, using the acceleration formula derived above.

“constant” acceleration of ship \[ a = 5.96 \times 10^{-2} \text{ m/s}^2 \]

What is the strategy?

1. Use a motion equation to find the speed of the ship at Mars’ orbit.

Physics principles and equations

In this calculation we will use the following motion equation.

\[ v_f^2 = v_i^2 + 2a \Delta x \]

Step-by-step solution

We know the acceleration. The distance \( \Delta x \) between the orbits of Earth and Mars can be calculated from the orbital data in the table above. This is the information we need in order to calculate the final speed at Mars’ orbit.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. [ v_f^2 = v_i^2 + 2a \Delta x ]</td>
<td>motion equation</td>
</tr>
</tbody>
</table>
| 2. \[ \Delta x = 2.28 \times 10^{11} \text{ m} - 1.50 \times 10^{11} \text{ m} \]  
\[ \Delta x = 0.78 \times 10^{11} \text{ m} \] | difference of orbits |
| 3. \[ v_f^2 = 2(5.96 \times 10^{-2} \text{ m/s}^2)(0.78 \times 10^{11} \text{ m}) \]  
\[ v_f^2 = 0.930 \times 10^{10} \text{ m}^2/\text{s}^2 \]  
\[ v_f = 96,400 \text{ m/s} \] | evaluate |

As we have observed, the pressure of sunlight on the ship’s sail is not really constant. It diminishes as your distance \( r \) from the Sun increases. As the pressure of sunlight diminishes, so does the force on the sail, and the acceleration of the spacecraft. In order to take this decrease fully into account, you would need to use calculus.

The correction for nonconstant acceleration is negligible for the modest distance traveled during the first hour of flight. For the trip to Mars’ orbit, the correction is larger. In a computation not shown here, we used calculus and found that the actual final velocity of the space yacht is 97,300 m/s. This is greater than the figure yielded by the calculation above, which assumed a constant mean acceleration, but by less than 1%. The time it takes the spacecraft to reach the orbit of Mars (which was not asked for in this problem) is a little more than 16 days.

We also significantly simplified this problem by ignoring the effects of gravity. In fairness to science fiction writers who write about such craft, they typically describe them as being assembled in regions of space safely distant from major sources of gravity. They also often describe how the gravitational forces of planets and stars could be exploited to accelerate ships in their desired directions of travel.
34.15 - Interactive checkpoint: carrier waves

The aircraft carrier USS Enterprise has a topside area of 13,600 m$^2$, the average albedo of its upper surface is 0.380, and the intensity of the sunlight falling on it is 500 W/m$^2$.

What is the downward force exerted on the ship by the Sun?

The density of seawater is 1030 kg/m$^3$. How many cubic centimeters of water must the carrier displace to buoy it up against this force?

Answer:

\[ F = \ \text{N} \]

\[ V = \ \text{cm}^3 \]

34.16 - How electromagnetic waves travel through matter

Light and other forms of electromagnetic radiation can travel through a vacuum, and it is often simplest to study them in that setting. However, radiation can also pass through matter. If you look through a glass window, you are viewing light that has passed through the Earth’s atmosphere and the glass. Other forms of radiation such as radio waves pass through matter, as well.

This section focuses on how such transmission occurs. It relies on a classical model of electrons and atoms that predates quantum theory. In this model, electrons orbit an atom. They have a resonant frequency that depends on the kind of atom. On a larger scale, atoms themselves and the molecules composed of them also have resonant thermal frequencies at which they can vibrate or rotate.

We will use the example of light striking the glass in a window to discuss how substances transmit (or do not transmit) electromagnetic radiation. When an electromagnetic wave encounters a window, it collides with the molecules that make up the glass. If the frequency of the wave is near the resonant thermal frequency of the glass molecules, which is true for infrared radiation, the amplitude of the molecules’ vibrations increases. They absorb the energy transported by the wave, and dissipate it throughout the glass by colliding with other molecules and heating up the window. Because it absorbs so much infrared energy, the glass is opaque to radiation of this frequency, preventing its transmission.

Scientists in the 19th century noted a phenomenon in greenhouses caused by the opaqueness of glass to infrared radiation, which they called the greenhouse effect. The glass in a greenhouse admits visible light from the Sun, which is then absorbed by the soil and plants inside. They reradiate the solar energy as longer infrared waves, which cannot pass back out through the glass and so help warm up the greenhouse. The same phenomenon occurs on a vaster scale in the atmosphere as gases like methane and carbon dioxide trap solar energy near the Earth’s surface.

In contrast to infrared radiation, higher frequency radiation such as visible light does not resonate thermally with atoms or molecules, but may resonate with the electrons of the atoms of a substance. In glass, visible light experiences much less reduction in the amplitude of its waves than infrared radiation does, and most of its energy passes through the glass quite easily. Atoms with resonant electrons that do absorb energy from a light wave quickly pass on that energy by re-emitting it as radiation of the same frequency to other atoms, which in turn pass it on to their neighbors.

This chain of absorptions and re-emissions, called forward scattering, follows a path close to the light’s original direction of travel. A beam of light that strikes a pane of glass will reach the “last atom” on the far side of the pane in an extremely short time. We see the light after it emerges, and think of glass as transparent.
This process does slow the transmission of the wave, which is why light travels slower in glass than it does in air or a vacuum (a fact captured numerically by the index of refraction of glass). For instance, light travels through a typical piece of optical glass at about 2/3 of its speed in a vacuum. Of course, 2/3 of the speed of light in a vacuum is still a rather rapid pace….

If atoms of certain substances, such as cobalt, are added to glass, they may absorb certain frequencies of light without re-emitting them. Cobalt glass has a deep blue-violet color, which indicates that all the lower visible frequencies (from red through green) are absorbed and cannot pass through it. Substances which absorb all frequencies of visible light are called opaque.

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**Polarized wave:** A transverse wave that oscillates in a single plane.

**Polarized radiation:** A form of radiation in which the electric field of every wave oscillates in the same plane.

Polaroid sunglasses like the ones shown above reduce the amount of light that passes through them. How do they do it? They only let through light waves whose electric fields oscillate in a certain plane. When a source such as the Sun emits light, waves emerge whose electric fields vibrate in every plane parallel to the direction of propagation. This is shown in Concept 1. The electric field of each individual wave does oscillate consistently in its own plane, which is called the wave’s plane of polarization, but the radiation as a whole does not have this property.

The electric field of any electromagnetic wave is a vector quantity. A polarizing lens or filter works by only letting through a certain component of the electric field of every light wave that strikes it. An ideal polarizing filter can be visualized as a set of narrow parallel slits whose direction is the filter’s transmission axis.

When a wave passes through the filter, the component of its electric field parallel to the transmission axis is what passes through. The component perpendicular to the axis is absorbed. As a result, waves that oscillate parallel to the slits pass through unhindered, while waves that oscillate perpendicularly to them are completely absorbed. You may want to consider an analogy: a rope passing through a gap in a picket fence. If you shake the rope vertically, the wave you create passes through unhindered. If you shake it horizontally, the wave collides with the pickets, transfers energy to them, and does not pass through. In the case of a wave passing through at an oblique angle, the component of its transverse displacement along the “picket axis” passes through while the fence absorbs the other component.

This basic model of how polarization works is shown with an ideal polarizing filter in Concept 2. In the diagram, the transmission axis happens to be vertical. A “slit” allows waves oscillating in a vertical plane to pass through. For waves that oscillate in other planes, only the vertical component of the electric field of the wave can pass through. Waves with a horizontal plane of polarization cannot get through the slit at all.

Ordinary light consists of waves whose electric fields are randomly oriented in all lateral directions. This is called unpolarized radiation. Two common sources of unpolarized light are the Sun and incandescent light bulbs. If the radiation is created or filtered so that it has only waves oscillating in a single plane, then it is linearly polarized. In the illustration in Concept 2, the polarizing filter is exposed to unpolarized radiation, and it transmits linearly polarized radiation.

In Concept 3 you see an end-on “close up” of several light waves striking a polarizing filter that has a vertical transmission axis. The light waves are coming toward you. In each case, the vertical component of the electric field of the light is transmitted, and the filter absorbs the horizontal component. The original field is shown as a solid, dimmed vector; its components are hollowed out. The vertical component that passes through is drawn with a bright color, and the horizontal component that is blocked is dimmed and marked with a red 

The filter transmits polarized light

- Incoming light is unpolarized
- Transmitted light vertically polarized
- Transmitted waves all in same plane

Ordinary light consists of waves whose electric fields are randomly oriented in all lateral directions. This is called unpolarized radiation. Two common sources of unpolarized light are the Sun and incandescent light bulbs. If the radiation is created or filtered so that it has only waves oscillating in a single plane, then it is linearly polarized. In the illustration in Concept 2, the polarizing filter is exposed to unpolarized radiation, and it transmits linearly polarized radiation.

The final illustration, in Concept 4, displays an experiment with two polarizing filters. Unpolarized light is coming toward you from a distant source. It passes first through the upper filter, which allows the passage of light that is polarized at the angle shown by the “slit” lines. This polarized light continues toward you and passes through the lower filter.

The left-hand part of the illustration shows the orientation of the filters, with parallel lines indicating the transmission axis of each filter. The right-hand side shows you the amount of light that passes through the area of overlap, and how that changes with the angles between the two axes. When the axes are perpendicular, no light at all can pass through.
Radiation also can be partially polarized, having a few waves oscillating in all planes, but with most of its waves concentrated in a single plane. This is true of sunlight scattered by the atmosphere. As the photo above shows, the sky in certain directions is partially polarized in a vertical plane so that most of its light can pass through a pair of sunglasses whose transmission axis is vertical. Less light (but still some) passes through the rotated sunglasses. (Polarizing sunglasses are specifically intended to reduce horizontally polarized glare reflected from roadways and water, not sunlight.)

Many forms of artificial electromagnetic radiation are polarized. A radio transmitter emits polarized radiation. If the rods of its antenna are vertical, then so is the electric field of every radio wave it creates. In this case, the most efficient receiving antenna is also vertically oriented; a horizontal receiving antenna would absorb radio waves much less efficiently. You may be familiar with this fact if you have ever tried to maneuver a radio antenna wire or a set of television “rabbit ears” to get the best reception. (If you do not know what “rabbit ears” are for television, well, before there was cable television, there was....)

E-field component passes through filter
If transmission axis is vertical, filter ...
· Transmits vertical E-field component
· Absorbs horizontal E-field component

Two Polaroid filters
Light passes through back (upper) filter
Then passes through front (lower) filter
Intensity depends on orientation of filters

---

34.18 - Polarization and intensity

A polarizing sheet, as found in some sunglasses, reduces the intensity of the light that passes through it. A common form of such material is called Polaroid. In this section, we discuss the structure of this material and then review its effect on intensity.

Edwin Land began studying this type of plastic when he was 17 and a student at Harvard University. He dropped out, returned to school, and then left again. In 1937 he and his colleagues formed the Polaroid Corporation, long famous for its instant cameras but well known for its sunglasses, as well.

Polaroid material consists of molecules that are long chains of atoms, stretched during manufacture so that they all line up in the same direction. Electrons can move freely along these chains and are able to absorb the energy of electric fields oscillating in the direction of the chain. The result is that electromagnetic waves whose electric fields are parallel to the Polaroid molecules are absorbed, while those with perpendicular fields pass through freely. This means that the transmission axis of the material is perpendicular to the direction of alignment of the molecules, as indicated in the diagram above.

In the previous section, we showed what happens qualitatively when unpolarized light, coming toward you, passes through two polarizing filters. This experiment is shown again in Equation 1. The top filter, through which the light passes first, is called the polarizer: The light is polarized after passing through this filter. The bottom filter, through which the light passes subsequently, is called the analyzer.

When the transmission axes of the filters are parallel, the polarized light from the polarizer has no trouble passing through the analyzer. As the analyzer rotates, the amount of light able to pass through both filters steadily decreases. When the transmission axes of the filters are at right angles, a configuration referred to as “crossed,” no light gets through at all. At this point, all the polarized light coming from the polarizer is perpendicular to the transmission axis of the analyzer and it gets completely absorbed there.

The results of this experiment can be expressed by two equations, which you see in Equation 1. The angle $\theta$ is measured between the transmission axis of the analyzer and the axis of the polarizer behind it. The light from the polarizer has an electric field with amplitude $E_p$. The component of this electric field parallel to the analyzer’s transmission axis is $E_p \cos \theta$. This component, which we have designated $E_a$, passes through the analyzer.

The equation should confirm how you would expect two polarizing materials to function. The polarizer allows electric fields with a specific orientation to pass through. If the angle between the transmission axes of the polarizer and the analyzer is zero (the axes are parallel) then this
electric field passes through undiminished. Mathematically, the cosine of zero is one, so the equation confirms this analysis. Conversely, if the axes are perpendicular ($\theta = 90^\circ$) then no wave can pass through both filters, and the amplitude of the electric field of the doubly filtered wave is zero. The cosine of $90^\circ$ is zero, so again the equation confirms what a qualitative understanding of polarization would lead you to believe.

It is also possible to express the intensity $I_a$ of the light transmitted by the analyzer in terms of the intensity $I_p$ of the polarized light coming to it from the polarizer. This is the second equation you see in Equation 1. It is named after the French engineer Louis Malus (pronounced “mah-LOO”), who discovered it. The law is a direct consequence of the equation above it, since $I = E^2/2\mu_0c$.

Equation 2 provides a formula for calculating the intensity of unpolarized light as it passes through a single polarizing filter. The incoming light has waves whose electric fields, of amplitude $E_0$, are randomly directed, making various angles $\theta$ with the transmission axis of the filter. The resulting transmitted intensity for each individual wave is $I_0 \cos^2 \theta$. The overall transmitted intensity is the average, over all angles $\theta$, of the intensities of these waves. Since the average value of a squared cosine function over all angles is exactly $\frac{1}{2}$, the intensity after the polarizing filter is half the original intensity.

That is, when unpolarized light passes through a filter, the intensity of the transmitted polarized light is one-half the intensity of the incident unpolarized light. For this reason, Polaroid sunglasses, which are designed to completely block horizontally polarized glare, at the same time have the effect of cutting in half the overall intensity of ambient, unpolarized daylight reaching the eyes.

The example problem asks you to obtain a surprising result: More light may pass through three filters than two. Two polarizing filters crossed at $90^\circ$ transmit no light at all. In the problem, we introduce a third, middle filter whose transmission axis is “halfway between” the others, at an angle of $45^\circ$. If this middle filter is inserted between the polarizer and the analyzer, the amount of light passing through the analyzer actually increases!

This is because when polarized light from the polarizer encounters the middle filter, the component of each wave that is parallel to the middle filter’s transmission axis passes through. So, the light passing through the combination of the first two filters is diminished (but not stopped), and it has a polarization angle of $45^\circ$. This polarized light encounters another “$45^\circ$ obstacle” when it reaches the analyzer, which it passes through diminished again, but not stopped. In this way, polarizing filters behave quite differently than dye or pigment filters, which always have a strictly subtractive effect on the transmission of light passing through them.
The “middle” filter is inclined at 45°. If it is placed between the “crossed” polarizer and analyzer filters, what is the new value of \( I_a \)?

For polarizer-to-middle: \( \theta = 45° \)

\[
I_m = I_p \cos^2(45°)
\]

\[
I_m = I_p / 2
\]

For middle-to-analyzer: \( \theta = 45° \)

\[
I_a = I_m \cos^2(45°)
\]

\[
I_a = (I_p / 2) / 2 = I_p / 4
\]

In terms of unpolarized light \( I_0 \),

\[
I_a = (I_0 / 2) / 4 = I_0 / 8
\]

### 34.19 - Sample problem: gazing at the Sun

Light that delivers more than 500 µW of power to the human eye can severely damage the retina. Is it safe to stare at the Sun from Mars as long as you are wearing Polaroid sunglasses?

Mars’ atmosphere is so thin that you can ignore its effect on the intensity of sunlight. Assume that the pupil admitting light into the human eye is a circle with a diameter of 2.00 mm.

#### Variables

- Intensity of sunlight at Mars’ orbit: \( I_0 \)
- Radius of Mars’ orbit: \( r = 2.28 \times 10^{11} \text{ m} \)
- Power emitted by the Sun as light: \( P_s = 3.91 \times 10^{26} \text{ W} \)
- Intensity transmitted by polarizer: \( I_p \)
- Power delivered into eye: \( P \)
- Diameter of pupil: \( d = 2.00 \times 10^{-3} \text{ m} \)
- Area of pupil: \( A \)

#### What is the strategy?

1. Use the power \( P_s \) of the Sun to find the intensity \( I_0 \) of its visible electromagnetic radiation (white light) at the distance of Mars’ orbit.
2. Calculate the intensity \( I_p \) of the Sun’s visible light that passes through a polarizing filter on Mars.
3. Calculate the power \( P \) of the polarized sunlight that passes through an area the size of the pupil of a human eye on Mars.

#### Physics principles and equations

For an omnidirectional source of electromagnetic radiation,

\[
I = \frac{P}{4\pi r^2}
\]
For unpolarized light of intensity $I_0$ that strikes a polarizing filter, the transmitted intensity $I_p$ is

$$I_p = \frac{I_0}{2}$$

The intensity of electromagnetic radiation is measured in W/m$^2$. The power $P$ absorbed by an area $A$ illuminated by light of intensity $I$ equals the product $IA$.

**Step-by-step solution**

In the first steps we find an expression for the intensity $I_p$ of the sunlight passing through a polarizing filter at the distance of Mars from the Sun. We also state an expression for the area of the pupil of the human eye.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$I_0 = \frac{P_s}{4\pi r^2}$</td>
</tr>
<tr>
<td>2.</td>
<td>$I_p = \frac{I_0}{2}$</td>
</tr>
<tr>
<td>3.</td>
<td>$I_p = \frac{P_s}{8\pi r^3}$</td>
</tr>
<tr>
<td>4.</td>
<td>$A = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$</td>
</tr>
</tbody>
</table>

We use the expressions derived above to find an expression for the power of the polarized sunlight entering the human eye on Mars, and in the last step we evaluate this expression.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>$P = I_p A$</td>
</tr>
<tr>
<td>6.</td>
<td>$P = \left(\frac{P_s}{8\pi r^3}\right) \left(\frac{\pi d^2}{4}\right)$</td>
</tr>
<tr>
<td>7.</td>
<td>$P = \frac{P_s d^2}{32 r^2}$</td>
</tr>
<tr>
<td>8.</td>
<td>$P = \frac{(3.91 \times 10^{-6} \text{ W})(2.00 \times 10^{-3} \text{ m})^2}{32(2.28 \times 10^{11} \text{ m})^2}$</td>
</tr>
</tbody>
</table>

As you can see, the power of the electromagnetic radiation entering the observer’s eyes substantially exceeds the safety limit of 500 $\mu$W. Even with Polaroids, the Sun is too bright to look at from the distance of Mars, although the Red Planet is half again as far from the Sun as the Earth is. As for the dubiously authentic experiment depicted above: Do not try this on your home planet!

---

**34.20 - Interactive checkpoint: the polarized sandwich**

Unpolarized light having intensity $I_0$ strikes a polarizer filter, then passes sequentially through three analyzer filters $A_1$, $A_2$, and $A_3$. These filters are rotated clockwise through the angles shown with respect to the orientation of the polarizer. What is the intensity $I_3$ of the light transmitted by $A_3$, in terms of the original intensity $I_0$?

Answer:

$$I_3 = \boxed{\frac{I_0}{2}}$$
Scattering: Absorption and re-emission of light by electrons, resulting in dispersion and some polarization.

The answer to a classic question – Why is the sky blue? – rests in a phenomenon called scattering. In this section, we give a classical (as opposed to quantum mechanical) explanation of how scattering occurs.

When light from the Sun strikes the electrons of various atoms in the Earth’s atmosphere, the electrons can absorb the light’s energy, oscillating and increasing their own energy. The electrons in turn re-emit this energy as light of the same wavelength. In effect, the oscillating electrons act like tiny antennas, emitting electromagnetic radiation in the frequency range of light.

An electron oscillates in a direction parallel to the electric field of the wave that energizes it, as shown in Concept 1. The electron then emits light polarized in a plane parallel to its vibration. We show a particular polarized wave that is re-emitted downward toward the ground, since we are concerned with what an observer on the surface of the Earth sees. Other light is scattered in other directions, including light scattered upward and light scattered forward in its original direction of travel.

Scattering explains why we see the sky: Light passing through the atmosphere is redirected due to scattering toward the surface of the Earth. In contrast, for an astronaut observer in the vacuum of space, sunlight is not scattered at all so there is no sky glow: Except for the stars, the sky appears black. To the astronaut, the disk of the Sun, a combination of all colors, looks white. We illustrate this below: The full spectrum combines to form white light.

The question remains, why is our sky blue rather than some other color? Light at the blue end of the visible spectrum, which has the shortest wavelength, is 10 times more resonant with the electrons of atmospheric atoms than red light. This means blue light is scattered more than red, so that more of it is redirected toward the ground.

Scattering also explains why we see the Sun as yellow rather than white. When you look up at the disk of the Sun from the Earth’s surface, the bluest portion of its light has been scattered away to the sides. The remaining part of the Sun’s direct light appears somewhat yellowish.

You may also have noted how the Sun appears to change color when it sets. As the Sun’s disk descends toward the horizon, its light must pass through a greater and greater thickness of atmosphere in order to reach you. Since a certain amount of sunlight is scattered aside for each kilometer of atmosphere it passes through, its position at sunset causes it to lose large amounts of light at the blue end and even toward
the middle of the visible light spectrum. At sunset, practically all the shorter wavelengths of light have been scattered out of it, leaving only light at the red end of the spectrum to be viewed by you. The “missing” blue light is not really missing. People to the west of you perceive it as the daytime sky.

Scattering also explains why skylight is partially polarized. When the Sun is low in the sky, as depicted in Concept 1, horizontally polarized light that gets scattered down from the overhead sky is polarized in the plane of the downgoing wave you see in the illustration. Incoming sunlight that is polarized in other planes also gets scattered, but not straight down towards the ground.

You can experiment with skylight polarization yourself if you have a pair of polarizing sunglasses. In the early morning or late afternoon hold your glasses against the northern or southern sky at arm’s length. Turn one of the lenses slowly, recalling that its transmission axis is vertical when the sunglasses are worn normally. You will find that the skylight is partially polarized in a plane perpendicular to the direction to the Sun.

**Optically active substance**: One that rotates the plane of polarization of light passing through it.

Substances that change the direction of polarization of light passing through them are called optically active. In this section we will discuss applications of optical activity in solids and liquids: stress analysis and polarimetry.

Certain transparent substances, such as the plastics Lucite® and Plexiglas®, are not normally optically active, but they become so when they are stressed. The greater the resulting strain in the plastic, the more it rotates polarized light; more precisely, the more it rotates the plane of polarization of the light. These kinds of materials possess an optic axis, which behaves quite differently from a transmission axis. The speed of polarized light passing through them depends on the orientation of the light’s plane of polarization with respect to the optic axis, and the plane of polarization is gradually rotated toward or away from the optic axis as the light passes through.

When a stressed plastic is placed between a polarizer and a crossed (perpendicular) analyzer and illuminated, the transmitted light will be zero where there is no strain (no rotation occurs) and brightest where there is the most strain (the most rotation occurs). The longer the wavelength of light, the less its plane of polarization rotates for a given strain, and the more strain it takes to rotate it through an angle that will allow it to pass through the analyzer. An example of this is shown in Concept 1: In the plastic model, the red regions are showing the most strain, while the violet and blue regions are showing the least strain.

The hands in the photograph, which are illuminated from behind, appear black. The polarized background would normally appear black too; in this photograph it has been brightened by the addition of extra cross-polarized light that is able to pass through the analyzer.

Engineers often use the optical activity of transparent solids in a procedure called optical stress analysis. Before building a bridge, for example, they may build a plastic model of the bridge and subject it to various loads. By using polarized light to observe how the strains imposed by a load distribute themselves over the structure, they can discover which parts of it are the most vulnerable to stress.

Many substances, when dissolved in a liquid such as water or alcohol, form a solution that rotates polarized light. For example, if a beam of polarized light passes through a solution of sugar water, its plane of polarization may be rotated either clockwise or counterclockwise. A solution of grape sugar, or dextrose, rotates polarized light to the right (clockwise). In fact, its name comes from the Latin word for right. The more healthful fruit sugar fructose, sometimes called levulose from the Latin word for left, rotates polarized light in the opposite direction.

Why does this rotation occur? When a light wave is absorbed and re-emitted by an electron in a dissolved asymmetrical molecule such as dextrose, the forward scattering imparts a slight twist to its plane of polarization. Optically active molecules in a solution contain complex asymmetric electric fields that can change the plane of polarization of a wave.

The greater the number of molecules a light wave interacts with in an optically active substance, the more its plane of polarization rotates. For this reason, the overall rotation of polarized light passing through a solution depends on the length of the light path through the solution and on the concentration of the optically active substance.
the concentration of the dissolved substance. The rotation is also proportional to a constant \( \alpha_0 \), called the specific rotation of the substance, which reflects the rotating power of its molecules. These relationships are summarized to the right in the polarimeter equation. Note that (and this is unusual for a physics equation) the rotation angle \( \alpha \) is measured in degrees rather than radians, and the clockwise direction is considered positive.

The polarimeter is a device that can be used to measure the net rotation of polarized light passing through an optically active solution. An experimenter directs polarized light through a container of the solution to be analyzed. The analyzer, which starts out parallel to the polarizer, does not transmit all the light from the polarizer because the light’s plane of polarization has been rotated by the solution. The experimenter turns the analyzer to one side or the other until the transmitted light has maximum brightness. Then she knows that the analyzer’s transmission axis matches the rotated polarized light, and she can measure the angle \( \alpha \) through which the analyzer has turned.

The polarimeter equation gives an expression for the angle \( \alpha \) of the analyzer at which the transmitted light will be the brightest. If the polarized light encounters more molecules of the optically active substance, either because the solution is more concentrated or because the immersed light path is longer, the rotation will be greater. Since the amount of rotation also depends on the wavelength of the light, the specific rotations \( \alpha_0 \), given in tables for particular dissolved substances are based on a polarimeter employing the 589 nm light that is emitted by a sodium vapor lamp.

Dextrose and fructose molecules are chemically identical (they have the same atoms arranged in the same pattern) but they are mirror images of each other. Because of this they rotate polarized light by the same amount in opposite directions. Organic molecules such as carvone may exist in two mirror image forms; you smell carvone as caraway or spearmint, depending on which way the molecule twists. The scents are different because the smell receptors in the nose react differently to the mirror image forms.

Using a polarimeter is one way to distinguish between the two forms of mirror image compounds. Also, if the specific rotation of a particular substance is known, the device can be used together with the polarimeter equation to determine the concentration of the substance in a solution. You are asked to perform such an analysis in the example problem to the right.

The diagrams below show the mirror image molecular forms of the citrus oil limonene, which is the essence of either orange or lemon, depending on the orientation of its molecules! (The gray spheres represent carbon atoms, and the blue spheres are hydrogen atoms.)

---

**Polarimeter equation**

\[
\alpha = \frac{dc\alpha_0}{100}
\]

- \( \alpha \): rotation of light (° clockwise)
- \( d \): length of immersed light path (m)
- \( c \): concentration of substance (kg/m³)
- \( \alpha_0 \): specific rotation of substance (°m²/kg)

---

**Example**

Carvone’s specific rotation is +62.5 (caraway) or –62.5 (spearmint). What is the concentration of the carvone in this beaker?

\[
c = \frac{100\alpha}{d\alpha_0}
\]

Counterclockwise rotation means spearmint, so we use \( \alpha_0 = -62.5 \):

\[
c = \frac{100(-62.5)}{(0.12)(-35°)}
\]

\[
c = 470 \text{ kg/m}^3
\]

---

34.23 - Physics at work: liquid crystal displays (LCDs)

The liquid crystal display (LCD) in the watch face above demonstrates variable optical activity at work. LCDs are found in many common devices, including calculators, cellular telephones and clocks. There are two types of LCDs: backlit and reflective. The one shown above is a reflective LCD, but we will explore both types in this section. Backlit LCDs generate light behind their displays; reflective LCDs like the one above utilize ambient light.

LCDs rely on polarization. The characters of a digital watch display consist of “digit” segments: regions that can be made dark. These segments are filled with a substance called liquid crystal that is optically active.
in its natural state but becomes inactive when a potential difference is applied across it. You see this phenomenon illustrated for a backlit LCD in Concepts 1 and 2. In Concept 1, the power is off (there is no potential difference across the crystal), so the crystal is optically active and rotates the plane of polarization of the light. The thickness of the crystal is designed to rotate the light by 90°. In Concept 2 a potential difference is applied, the crystal becomes inactive, and there is no rotation of the light's plane of polarization.

The liquid crystal is sandwiched between two polarizing filters with perpendicular transmission axes, as shown in Concept 3. In the backlit LCD shown there, a light behind the display shines through the left-hand filter. Following our usual practice we will call this filter that is closer to the light source the polarizer, and the one farther away the analyzer. The polarizer allows light with a particular plane of polarization to pass through. When the liquid crystal is “on” (optically inactive), it does not rotate the polarized light, and the analyzer prevents any of the light from passing through, creating a dark area.

When no potential difference is applied across the liquid crystal, it is “off” and reverts to its normal optically active state. The perpendicular orientation of the analyzer now allows the polarized light to pass through. Instead of being black, the material of the segment looks the same as the adjoining material. In order to make the transmitted light difficult to see, the display’s background is colored to resemble it. This is shown in Concept 4.

If you examine the watch in the illustration above or one on your wrist, or a friend’s wrist, you will see that the digits it displays are pieced together from segments that appear black when they are “turned on.” When they are off, these segments are invisible to the casual glance, but they can still be made out as faint shadows if you look very carefully.

We have been describing a typical backlit LCD, which can be read in the dark. The backlighting consumes more power than any other part of the display, since the amount of power required to turn on each digit segment is negligible. Backlit LCDs are typically found in automobile dashboards, where their power usage is not a particular concern and where readability at night is important.

A reflective LCD, such as the one in the watch above, consumes less power. We show how a reflective LCD works in Concept 5: it is a bit more complicated than a backlit LCD. A mirror that can reflect ambient light coming from in front of the LCD replaces the backlight behind the polarizer. As with the backlit display, the transmission axes of the polarizer and analyzer are at right angles to each other.

Concept 5 shows what happens when a liquid crystal digit segment in this type of LCD is turned off. In this state, light passes through each of the analyzer, the liquid crystal segment, and the polarizer twice. Unpolarized light enters the display from the right and becomes horizontally polarized as it passes through the right-hand filter (the analyzer). It rotates 90° to a vertical orientation as it passes through the optically active liquid crystal component. In this orientation it can pass through the analyzer, reflect off the mirror, and come back out through the polarizer without hindrance. (A keen-eyed observer will note that the angular symmetry of reflection is apparently being violated where the light strikes the mirror: We drew the incident ray at a different angle in order to fit all the details into the diagram.)

After passing through the polarizer, the light retraces its path through the liquid crystal, again rotating and again becoming horizontally polarized. It passes back out through the analyzer without absorption, creating a region that blends into the background color of the display.

When the segment is turned on, it becomes optically inactive; optically, it can be treated as if it were no longer there. Light entering from the right gets horizontally polarized by the analyzer, and propagates to the left until it strikes the polarizer, where it is absorbed. This light never even reaches the mirror, and the segment appears dark. You see this happening in the watch above, where the dark segments spell out the time of day.

More elaborate LCDs of both types can be manufactured with “segments” having any shape, not just the parts of digits. For example, the battery and signal strength icons on a cellular telephone display, or the letters “H” and “M” on the watch above, use specially shaped segments. Some flat panel display screens use an array of thousands or even millions of tiny LCD dot-segments to produce virtually any image. With the appropriate refinements, color images can be produced.

As with polarized light from the sky, you can use a pair of Polaroid sunglasses to experiment with the polarized light emitted by a liquid crystal display. For example, you will find that a watch, viewed through the glasses, looks quite different depending on its orientation. At some angles the display looks more or less normal, while at others it becomes completely unreadable.
34.24 - Gotchas

A light wave is a transverse wave. Yes. Both of its components, an electric and a magnetic field, oscillate perpendicularly to its direction of travel.

Radio signals and light waves are fundamentally different. Both are forms of electromagnetic radiation, so we lean toward “no” in response to this statement. The wavelength and frequency of radio transmissions and light are significantly different, and humans can see light, but not radio waves, so one could say “yes”. However, both are electromagnetic waves, and both move at the speed of light.

Intensity is the same thing as average energy density. No, intensity represents the average rate of power transmission of an electromagnetic wave per unit area, perpendicular to the direction of propagation of the wave. Average energy density represents the average amount of energy contained in an electromagnetic wave per unit volume.

34.25 - Summary

An electromagnetic wave is a traveling wave consisting of mutually perpendicular electric and magnetic fields that oscillate transversely to the direction of propagation. Electromagnetic radiation moves at the “speed of light,” or $c$, which is 299,792,458 m/s in a vacuum. The value $3.00 \times 10^8$ m/s is often used.

Every electromagnetic wave has a characteristic frequency and wavelength. The electromagnetic spectrum is an ordering of electromagnetic radiation in accordance with these two properties and extends far beyond the tiny gamut called visible light that we can detect with our eyes. Some other kinds of electromagnetic radiation are radio waves (AM and FM), television signals, microwaves, infrared light, ultraviolet light, x-rays, and gamma rays.

Oscillating electric and magnetic fields propagate as a wave indefinitely, as the changing magnetic field induces a changing electric field (Faraday’s law), and the changing electric field in turn induces a changing magnetic field (Maxwell’s law of induction). An electromagnetic wave does not require a medium to propagate.

Electromagnetic plane waves are the form of electromagnetic radiation that is the simplest to analyze. With these waves, wave fronts advance through space in a series of parallel planes; they are approximated by radiation from a very distant point source. The electric and magnetic fields that constitute plane waves can be mathematically described by field equations that are sinusoidal functions of position and time. At a fixed point in space, both fields oscillate sinusoidally in time. At a fixed instant in space, both fields can be depicted as mutually perpendicular sinusoidal wave trains.

Maxwell’s equations can be used to derive a pair of general wave equations. These differential equations must be satisfied by any function of position and time that describes an electromagnetic wave. The sinusoidal field functions for a plane wave satisfy these general wave equations. The wave equations imply that the electric and magnetic field strengths of an electromagnetic wave are proportional at all points in space and at all times, with the constant of proportionality being $c$, the speed of light.

Maxwell proved that the speed of an electromagnetic wave equals the reciprocal of the square root of the product of two fundamental physical constants, the electric permittivity of free space $\varepsilon_0$ and the magnetic permeability of free space $\mu_0$. This value turned out to be equal to the empirically well-measured speed of light, providing strong evidence that light is a form of electromagnetic radiation.

The intensity of electromagnetic radiation equals the average power transported by the radiation per unit area, measured perpendicularly to the direction of propagation.

### Equations

<table>
<thead>
<tr>
<th>Proportionality of fields</th>
</tr>
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<tr>
<td>$E = \frac{c}{\mu_0 \varepsilon_0}$</td>
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<tr>
<th>Poynting vector</th>
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<tr>
<td>$S = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$</td>
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<table>
<thead>
<tr>
<th>Intensity of electromagnetic radiation</th>
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<tr>
<td>$I = \frac{1}{2\mu_0 c} \frac{E_{\text{max}}^2}{\mu_0 c}$</td>
</tr>
<tr>
<td>$I = \frac{P}{4\pi r^2}$</td>
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<tr>
<th>Energy density</th>
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<tbody>
<tr>
<td>$u_E = \frac{\varepsilon_0 E^2}{2}$, $u_B = \frac{B^2}{2\mu_0}$</td>
</tr>
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</table>

| $u = u_E + u_B = 2u_E = 2 \mu_0$ |
| $u_{\text{avg}} = \frac{\varepsilon_0 E^2}{2}$ |

<table>
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<tr>
<th>Momentum transferred by radiation absorption</th>
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<tbody>
<tr>
<td>$\Delta p = \frac{\Delta U}{c}$ for a blackbody</td>
</tr>
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</table>
Intensity is measured in \( \text{W/m}^2 \). For visible light, it corresponds roughly to what humans perceive as brightness. The energy density of a wave is the amount of energy in the wave per unit volume. Energy density is measured in \( \text{J/m}^3 \). Both intensity and energy density are proportional to the square of the electric field amplitude \( E_{\text{max}} \) of the wave.

Electromagnetic radiation emitted isotropically (equally in all directions) by a point source takes the form of spherical waves rather than plane waves. The intensity of these waves is proportional to the power of the source and inversely proportional to the square of their distance from the source.

Electromagnetic waves exert radiation pressure on any surface they illuminate, proportional to the intensity of the radiation. The pressure on a perfectly reflective surface is twice that on a perfectly absorptive surface.

Linearly polarized light consists of light waves whose electric fields all oscillate in the same plane. This type of light can be created by several methods, such as by passing unpolarized light, whose electric fields oscillate in many directions, through a polarizing filter. The direction of polarization that results is called the transmission axis of the filter. As a randomly polarized wave passes through such a filter, only the component of its oscillating electric field that is aligned with the transmission axis passes through.

Polaroid sunglasses take advantage of the fact that a polarizing filter can decrease the intensity of the light that passes through it, and the transmission axes of their lenses are oriented vertically to block the horizontally polarized glare reflected from the surfaces of roadways and bodies of water.

The scattering of light passing through a transparent substance is the absorption and re-emission of light waves of characteristic frequencies by atoms in the substance. Scattering in the atmosphere is responsible for the blue glow of the sky, the yellowish hue of the Sun, and the red color of sunsets.

Optically active substances rotate the plane of polarization of polarized light that passes through them.

Optical activity forms the technological basis of the liquid crystal displays (LCDs) used in many consumer electronic devices.
Chapter Assumptions

Unless stated otherwise, electromagnetic waves are assumed to be propagating as plane waves through a vacuum.

When converting light-years to meters, assume that a year has 365 days.

The speed of light in a vacuum is $3.00 \times 10^8$ m/s.

The Sun radiates energy at the rate of $3.91 \times 10^{26}$ W.

Conceptual Problems

C.1 In a phenomenon called **diffraction**, waves are able to bend around obstacles. For example, ocean waves can bend around a piling so that it does not cast a down-wave “shadow” of calm water. The shorter the wavelength of a wave, the less able it is to diffract around obstacles. You have experienced this yourself: You cannot see the people in a room down the hall, but you can hear them talking, corresponding to the fact that a typical wavelength of light is about ten million times shorter than that of a typical sound. If you live near the big city, but behind a rather large mountain, which of the urban radio broadcasts are you likely to receive the best: AM or FM? Explain.

- AM
- FM

C.2 Half of all the electromagnetic radiation reaching the Earth from the Sun lies in the visible spectrum that can be perceived by the human eye: ranging from extremes of 390 nm to 780 nm. This is a tiny portion of the whole electromagnetic spectrum, and it can't be just a coincidence. Why is so much of the Sun's radiation visible?

C.3 Electromagnetic radiation is called **polarized** if every individual electromagnetic wave in the radiation has its electric field oscillating in the same plane. Is the radio-wavelength radiation emitted from a dipole antenna polarized? Explain.

- Yes
- No

C.4 Clouds scatter light just as the atmosphere does. Since clouds consist mainly of microscopic water droplets of many sizes, they scatter all the wavelengths of visible light equally. The smallest droplets scatter the shortest wavelengths at the violet end of the spectrum, and larger droplets scatter light of longer wavelengths. Since all wavelengths are equally scattered by a cloud, it appears white. So does a jet’s condensation trail, or fog, or steam, or mist on a window.

Well, not quite white. In fact if you look at a real (cumulus) cloud, you will see that parts of it appear bright white, while other parts appear gray, and at the bottom of a thick cloud, almost black. What accounts for these variations?

C.5 It is early morning, and the Sun is rising (where else?) in the east. (a) What is the direction of polarization of the skylight directly overhead? (b) What is the direction of polarization of the skylight as you look toward the northern horizon?

(a) i. East-west
   ii. North-south
(b) i. Vertical
   ii. Horizontal

C.6 Now it is noon, and the Sun is directly overhead. (a) What is the direction of polarization of the skylight coming from the eastern horizon? (b) What is the direction of polarization of the skylight as you look toward the northern horizon?

(a) i. Vertical
   ii. Horizontal
(b) i. Vertical
   ii. Horizontal
C.7 Sunlight which reflects off smooth surfaces like those of roadways or standing water becomes horizontally polarized in the process. Polarizing sunglasses are designed to completely block this reflected glare, while at the same time they reduce the intensity of unpolarized ambient light by one-half. (a) If a sunglasses lens is to block reflected glare, what should the orientation of its transmission axis be? (b) LCD displays, such as flat panel computer display screens, and certainly automobile instruments and digital wristwatch faces, should be readable by people wearing polarized sunglasses. In order for them to be readable, what should the transmission-axis orientation of their front "analyzer" filters be? (Note: The conceptual diagrams of LCDs in the text may or may not have been drawn to reflect this requirement.)

(a) □ Vertical □ Horizontal
(b) □ Vertical □ Horizontal

Section Problems

Section 1 - The electromagnetic spectrum

1.1 The speed of light in a vacuum is exactly 299,792,458 m/s. This speed is sometimes used to provide a convenient yardstick for large astronomical distances. (a) A light-second is the distance light travels in one second. If the Moon is 3.84×10⁸ m from the Earth (center to center), how many light-seconds away is it? (b) A light-minute is the distance light travels in one minute. If the Earth is 1.50×10¹¹ m from the Sun, how many light-minutes away is it? (Your answer also represents the number of minutes it takes for light to reach the Earth from the Sun.) (c) A light-year is the distance light travels in one year. The Sun is 28,000 light-years from the center of the Milky Way galaxy. What is this distance in meters?

(a) ________ light-seconds
(b) ________ light-minutes
(c) ________ m

1.2 Nowadays many highway tunnels are equipped with AM "repeaters" so that motorists can listen to their AM radios while inside. Otherwise, AM radio cannot be received in the tunnel. FM repeaters are not usually necessary, since FM reception in tunnels is often acceptable without any special assistance. A typical tunnel is 5 meters high. (a) What is the wavelength of the electromagnetic radiation from an FM station broadcasting at 94.1 MHz, which penetrates easily into the tunnel? (b) What is the wavelength of the electromagnetic radiation from an AM station broadcasting at 940 kHz, which ordinarily cannot penetrate into the tunnel?

(a) ________ m
(b) ________ m

1.3 A certain radio station broadcasts electromagnetic radiation with a wavelength of one quarter mile (0.25 mi). (a) What is the frequency of the broadcast? (b) Is the station AM or FM?

(a) ________ Hz
(b) □ AM □ FM

1.4 The color to which the human eye is most sensitive is a yellowish green with a wavelength of approximately 550 nm. (For this "scientific" reason there was a brief vogue in the 1980s for making fire trucks this color, a fad that faltered when departments found that firefighters took little pride in non-red fire trucks and consequently maintained them poorly. In fact, the photographer of the accompanying picture was told that the truck will be painted red "within a month – red is traditional"). (a) What is the frequency of waves of this color? (b) How many wavelengths of this yellowish green light are there in the thickness of a human hair that is 110 µm in diameter? (c) How many wavelengths of the light are there in the width of a pencil, 6.00 mm?

(a) ________ Hz
(b) ________
(c) ________
The rather unappetizing confection you see in the photograph is a carrot strip that has been cooked in a microwave oven from which the rotating turntable was removed. When it is turned on, the oven uses a "magnetron" tube to generate standing microwaves in the cavity of the oven. No cooking heat is generated at the locations of the standing-wave nodes, and maximal heat (microwave "hot spots") is generated at the locations of the standing-wave antinodes. The manufacturer's label on the oven states that it operates at a frequency of 2450 MHz. (a) Based on this frequency, what is the wavelength of the microwaves generated inside the oven? (b) Based on your answer to part a, what is the distance between two antinodes of the standing microwaves? Give your answer to three significant figures. (c) Is the result in part b compatible with the measurement shown in the photograph?

(a) ______ cm
(b) ______ cm
(c) Circle Yes or No

Apollo 11 astronauts left an array of "corner mirrors" on the Moon, each one consisting of three mirrors arranged at mutual right angles like the corner of a box. A corner mirror has the property that light shined on it from any angle is reflected back exactly in the direction from which it came. Scientists subsequently directed a brief pulse of light from a powerful laser at the mirror assembly on the Moon, and measured the time required for it to travel to the array and return back to Earth: 2.56 s. How far away was the array of mirrors at that time?

_______ m

The human eye contains receptors for just three colors of light: red, green, and blue. All the colors we perceive in the world are mixed in our brains from different proportions of these three primary colors. Computer and video display screens, both CRTs and flat panels, take advantage of this fact by using color-generating elements of the same three colors to create the appearance of any imaginable color. An image on the screen is built of tiny dots of color called picture elements, or "pixels". (By default, the period at the end of this sentence consists of just one black pixel.) Each pixel has three different color levels associated with it, one level for each of red, green, and blue, with integer values from 0 (representing no color), to 255 (representing the brightest level of a color). For example, (R, G, B) = (255, 0, 0) represents the brightest pure red, (170, 170, 0) represents an equal mixture of red and green, without any blue, which comes out a sort of dark yellow, and (127, 127, 127) represents an equal mixture of all three primary colors, each one at half its maximum intensity: a color sometimes called "50% gray".

(a) If black is the "color" corresponding to a lack of visible light, what are the RGB values of a black pixel? (b) If white is the "color" corresponding to the brightest possible equal mixture of all colors, what are the RGB values of a white pixel? (c) How many different colors can a pixel be? (The answer is large, but enter it as a whole number.)

(a) (________,________,________)
(b) (________,________,________)
(c) ________ colors
1.8 The **brightness** of a computer screen color is a percentage value calculated by dividing the brightest of its three color levels by 255, the maximum possible brightness. For example, the brightness of the magenta hue (R, G, B) = (127, 0, 135) is 135/255 or 53%. What is the brightness of each of the following colors: (a) black (0, 0, 0); (b) brightest white (255, 255, 255); (c) chartreuse (132, 251, 67); (d) brown (114, 73, 32).

(a) ________ %  
(b) ________ %  
(c) ________ %  
(d) ________ %

1.9 Equally balanced computer screen colors like (R, G, B) = (0, 0, 0), (255, 255, 255), or (120, 120, 120) have no predominant hue, and they look black, white, or some shade of gray in between: The grays with greater brightness are closer to white, and the grays with less brightness are closer to black. A color like (104, 100, 100) will appear slightly red, but it is mostly the gray color (100, 100, 100) with only a small additional amount of red. Color specialists say that this kind of color is "unsaturated". Colors that are "saturated" have a rich predominant color with little gray mixed in. For example the RGB color (246, 15, 18) is a very saturated red. It only contains a slight constituent of gray, corresponding to its lowest-level component: (15, 15, 15).

The **saturation** of an RGB color equals the percentage of its brightest component color that does **not** form part of its gray constituent. If the brightest component of an RGB color has value \(N\), and the dimmest component has value \(n\), then its gray constituent is \((n, n, n)\) and its saturation equals \((N - n)/N\). For example, the saturation of the vibrant red mentioned above is \((246 - 15)/246 = 94\%\). (a) What is the saturation of the red-tinged gray color (124, 100, 100)? (b) What is the saturation of the dark green (33, 76, 45)? (c) What is the saturation of the light blue (213, 225, 255)?

(a) ________ %  
(b) ________ %  
(c) ________ %

1.10 A live outdoor radio broadcast is disrupted when someone fires a gun into the air right in front of the microphone. The sound of the shot is broadcast via radio waves to a communications relay satellite in a geosynchronous orbit 35,790 km directly overhead, and back down to the radio in a police car parked 85.0 m away from the microphone. The sound waves of the gunshot also travel directly through the air to the car, at a speed of 343 m/s. Which sound does a police officer in the car hear first, the broadcast sound or the direct sound?

- [ ] Broadcast  - [ ] Direct

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**Section 2 - Electromagnetic waves**

2.1 The diagram shows the electric and magnetic field components of an electromagnetic wave at a certain location and at a certain instant in time. Is the wave traveling toward you or away from you?

- [ ] Toward you  - [ ] Away from you
Section 3 - Proportionality of electric and magnetic fields

3.1 An electromagnetic wave in a vacuum has an electric field amplitude of 417 V/m. What is the magnetic field amplitude of this wave?

3.2 An electromagnetic wave in a vacuum has an instantaneous magnetic field strength of 7.50e−7 T at a certain point. What is the instantaneous electric field strength of the wave at this point?

3.3 Here is a pair of sinusoidal functions describing an electric and a magnetic field that vary in space and time.

\[ E = \left( 262 \text{V/m} \right) \cos \left( \left[ 3.57 \times 10^{-7} \text{rad/m} \right] x - \left[ 1.07 \times 10^{16} \text{rad/s} \right] t \right) \]
\[ B = \left( 8.74 \times 10^{-6} \text{T} \right) \cos \left[ \left[ 3.57 \times 10^{-7} \text{rad/m} \right] x - \left[ 1.07 \times 10^{15} \text{rad/s} \right] t \right] \]

They cannot be the field equations of an electromagnetic wave. Why not?

Section 6 - Electromagnetic energy: the Poynting vector

6.1 In the text it is stated that when an electromagnetic wave intersects a surface obliquely, the instantaneous area power density conveyed to the surface depends on the angle \( \theta \) between its area vector and the Poynting vector of the wave. The equation expressing this relationship is

\[ S_\theta = \mathbf{A} \cdot \mathbf{S} \]

where \( S_\theta \) is the oblique area power density, \( \mathbf{A} \) is the area vector, and \( \mathbf{S} \) is the Poynting vector. Since \( \mathbf{S} \) is itself a cross product, \( S_\theta \) can be written as

\[ S_\theta = \mathbf{A} \cdot \left( \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right) = \frac{1}{\mu_0} \mathbf{A} \cdot (\mathbf{E} \times \mathbf{B}) \]

The vector product \( \mathbf{A} \cdot (\mathbf{E} \times \mathbf{B}) \) is called a vector triple product.

In the chapter on rotational dynamics, a computational formula based on a determinant was given for the vector cross product:

\[ \mathbf{E} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ E_x & E_y & E_z \\ B_x & B_y & B_z \end{vmatrix} \]

For this problem, show that a determinant formula for the vector triple product is

\[ \mathbf{A} \cdot (\mathbf{E} \times \mathbf{B}) = \begin{vmatrix} A_x & A_y & A_z \\ E_x & E_y & E_z \\ B_x & B_y & B_z \end{vmatrix} \]

From this you can conclude that a general formula for the instantaneous area power density of an electromagnetic wave striking a surface at the oblique angle \( \theta \) is

\[ S_\theta = \frac{1}{\mu_0} \begin{vmatrix} A_x & A_y & A_z \\ E_x & E_y & E_z \\ B_x & B_y & B_z \end{vmatrix} \]

Section 7 - Intensity and energy density

7.1 An electromagnetic wave has an electric field amplitude of 320 V/m. (a) What is the maximum magnitude of the Poynting vector of this wave? (b) What is the average magnitude of the Poynting vector? (c) What is the intensity of the wave? (d) What is the root mean square of the electric field of the wave?

(a) _______ W/m^2
(b) _______ W/m^2
(c) _______ W/m^2
(d) _______ V/m
7.2 An electromagnetic wave in a vacuum has a maximum magnetic field amplitude of $5.67 \times 10^{-7}$ T. (a) What is the maximum electric field amplitude of the wave? (b) What is the average power per unit area associated with the wave?
   (a) _______ V/m
   (b) _______ W/m²

7.3 An electromagnetic wave has an electric field strength of $6.00 \times 10^4$ V/m and a magnetic field amplitude of $2.00 \times 10^{-4}$ T at a certain location. (a) What is the electric field energy density at this location? (b) What is the magnetic field energy density? (c) What is the total energy density?
   (a) _______ J/m³
   (b) _______ J/m³
   (c) _______ J/m³

7.4 What is the average total energy density in electromagnetic radiation that has an intensity of 475 W/m²?
   _______ J/m³

7.5 A neodymium laser emits a pulse of radiation, providing 125 TW of power for a duration of 1.00 ns. How much energy is contained in that pulse?
   _______ J

7.6 An electromagnetic wave has an electric field strength of 145 V/m at a point P in space at time $t$. (a) What is the electric field energy density at P? (b) A parallel-plate capacitor whose plates have area 0.106 m² has a uniform electric field between its plates with the same energy density as the answer to part a. What is the charge on the capacitor?
   (a) _______ J/m³
   (b) _______ C

7.7 An electromagnetic wave has a magnetic field strength of $6.78 \times 10^{-6}$ T at a point P in space at time $t$. (a) What is the magnetic field energy density at P? (b) A 0.410 m long solenoid consisting of 950 loops of wire has a uniform magnetic field inside its coil with the same energy density as the answer to part a. The radius of the coil is 0.215 cm. What is the current passing through the solenoid?
   (a) _______ J/m³
   (b) _______ A

7.8 The instantaneous electric field energy density of an electromagnetic wave is $u_E = \varepsilon_0 E^2/2$. The instantaneous magnetic field energy density of an electromagnetic wave is $u_B = B^2/2\mu_0$. Show that $u_E = u_B$ at any instant in time.

7.9 The instantaneous electric field energy density of an electromagnetic wave is $u_E = \varepsilon_0 E^2/2$. The instantaneous magnetic field energy density of an electromagnetic wave is $u_B = B^2/2\mu_0$. (a) What is the average electric field energy density over time, expressed in terms of the electric field amplitude, $E_{\text{max}}$? (b) What is the average magnetic field energy density over time, expressed in terms of the magnetic field amplitude $B_{\text{max}}$? (c) Show that the average total energy density of the wave is $u_{\text{avg}} = \varepsilon_0 E_{\text{max}}^2/2$.

Section 10 - Radiation intensity and distance

10.1 The SMART-1 lunar orbiter, launched by the European Space Agency, reached the Moon in November 2004 after a 13-month voyage from the Earth. It used an ion propulsion system powered by two winglike solar panel assemblies, one of which you see deployed for testing in the photograph. The assembly consists of lithium photovoltaic cells and it measures 14 m wide by 115 cm tall. In flight, when the panels are adjusted to directly face the Sun, each assembly produces 1.9 kW of power. It was shown in the text that the intensity of sunlight is 1380 W/m² at the distance of the Earth's (and the Moon's) orbit. What is the efficiency, expressed as a percentage, of the SMART-1 solar panels at converting sunlight power into electric power?
   _______ %

10.2 Sunlight falling on the upper layers of the Earth's atmosphere has an intensity of 1380 W/m². If all the energy were carried by a single wave, (a) what would be the electric field amplitude $E_{\text{max}}$ for this wave? (b) What would be the magnetic field amplitude $B_{\text{max}}$?
   (a) _______ V/m
   (b) _______ T
10.3 Suppose a blue-giant star is located 10.7 light-years from the solar system and radiates energy at a rate 135 times that of the Sun. What would be the intensity of the starlight reaching the Earth from this star? (Treat this as an ideal problem, and ignore any absorption due to interstellar dust or the Earth's atmosphere.)

\[ \text{Intensity} = \text{Power} / \text{Area} \]

W/m²

10.4 The Andromeda galaxy pictured here is a vast whirl of \(1.0 \times 10^{11}\) stars, located 2.9 million light-years from the Earth. Assume that, on average, each star in this galaxy emits radiation with the same power as the Sun. (a) What is the intensity of light from this galaxy as it reaches the Earth? (Treat this as an ideal problem and ignore any absorption due to interstellar dust or to the Earth's atmosphere.)

(b) A candle emits radiation in all directions with a power of 4.2 W. What is the intensity of the candlelight at a distance of 9.3 km? (c) From which source is the light more intense, the galaxy or the candle?

(a) \[ \text{Intensity} = \text{Power} / \text{Area} \]

W/m²

(b) \[ \text{Intensity} = \text{Power} / \text{Area} \]

W/m²

(c) \[ \text{The galaxy} \quad \text{The candle} \]

10.5 The Voyager spacecraft was launched in 1977 on a mission to explore Jupiter, Saturn, and the reaches of interstellar space beyond them. It is by far the most distant human-made object in the universe: In September 2004 this craft was 14 billion kilometers from the Earth. Voyager uses a directional antenna that broadcasts radiation in a solid angular swath that covers 1.2% of its sky, with a power of 23 W. What was the intensity of its signal at the Earth in September 2004? (Find the maximum possible intensity by ignoring the attenuating effects of the Earth’s atmosphere and interstellar dust.)

\[ \text{Intensity} = \text{Power} / \text{Area} \]

W/m²

10.6 The radio telescope in Arecibo, Puerto Rico has a diameter of 300 m, and is able to detect radio signals with a power of as little as \(6 \times 10^{-22}\) W. If the alien inhabitants of a planet 100 light-years away are trying to contact us, what is the minimum power their transmitter must have if its signal is to be detectable at Arecibo? Assume that the aliens transmit omnidirectionally. State your answer to two significant digits.

\[ \text{Power} = \text{Intensity} \times \text{Area} \]

W

10.7 A tiny source is emitting electromagnetic radiation equally in all directions. At a distance of 1.0 cm this radiation can be considered to be a plane wave over a small enough region. At a point P 1.0 cm from the source, the electric field amplitude of the wave is 3.5 V/m. (a) What is the amplitude of the magnetic field of the wave at P? (b) What is the average intensity of the radiation at P? (c) What is the power with which the source emits radiation?

(a) \[ E = \text{Electric field amplitude} \]

(b) \[ \text{Intensity} = \text{Power} / \text{Area} \]

W/m²

(c) \[ \text{Power} = \text{Intensity} \times \text{Area} \]

W

10.8 A 40 W electric light bulb emits 3.25% of its consumed energy as blue light with a wavelength of 433 nm. Assume this energy is contained in a single electromagnetic wave, and write the plane wave field equations that approximately describe the light from this bulb at a distance of 12.5 m.
Section 12 - Intensity and field strength around a dipole antenna

12.1 Due to its high electrical conductivity, seawater weakens ordinary radio waves rapidly as they pass through it. Extremely low frequency (ELF) radio waves in the range from 40–80 Hz are the least subject of all radio waves to this kind of attenuation, and so are an attractive option for communicating with cruising submarines. Signals generated at a land-based naval station are broadcast toward the sky, where they bounce off the ionosphere (an electrically conductive layer high in the atmosphere), and then propagate downward to penetrate the sea and reach their recipients.

One of the difficulties associated with implementing ELF is the problem of generating a signal in the required frequency range. (a) What is the wavelength of an ELF radio wave having a frequency of 75 Hz? (b) How long is a half-wave dipole antenna that broadcasts waves of this frequency? (c) An AC-driven linear antenna with the power feed at one end rather than in the middle is called a quarter-wave dipole. How long is a quarter-wave dipole antenna that broadcasts radio waves with a frequency of 75 Hz?

(The longest ELF antenna ever constructed is a horizontal pole-mounted wire constructed by the U.S. Navy in Wisconsin, 222 km long. It is neither a half-wave nor a quarter-wave dipole, and uses technological refinements to achieve signals in the desired frequency range.)

(a) ________ m
(b) ________ m
(c) ________ m

Section 13 - Radiation pressure

13.1 A regulation NFL football field, including its end zones, is exactly 120 yards long and 160 feet wide. At noon on a certain cloudless day, the Sun is shining on the field with an intensity of 825 W/m². The albedo of the field is 0.550. (a) What is the downward force of sunlight on the field? (b) A newly minted half-dollar coin weighs 11.340 g. Which force is greater: that exerted by the light pressure on a football field, or the weight of two half dollars still sitting on it after several coin tosses by a clumsy referee?

(a) ________ N
(b) i. The force due to light pressure
   ii. The weight of the coins

13.2 A laser pointer directs a narrow beam of light that can be used by a speaker to illuminate locations on a projected image during a lecture. If a 3.7 mW laser pointer illuminates a circular area with a radius of 1.2 mm on a projection screen with an albedo of 0.86, find the pressure the laser exerts on the illuminated portion of the screen.

    ________ N/m²

13.3 (a) What is the radiation pressure that a 250 W light bulb emitting light with an efficiency of 15% exerts on a small matte-black surface at a distance of 0.500 m? (b) What is the pressure it exerts on a small mirror at the same distance?

(a) ________ N/m²
(b) ________ N/m²

13.4 The Echo 2 satellite, a perfectly reflective aluminized Mylar® sphere 41.0 meters in diameter, was launched in 1964 into an orbit a little more than 1000 km above the Earth's surface. It was an experimental “passive communications” satellite, serving to bounce radio signals from a ground transmitter to a receiver a quarter of the globe away. As the first human-made object visible from space, it inspired countless young future physicists with its visible progress as a bright star drifting across the heavens. (a) The distance of Echo 2 from the Sun was 1.50×10¹¹ m, and assume the the power emitted by the Sun is 3.91×10²⁶ W. What was the intensity of sunlight striking the satellite? (b) What pressure did the sunlight exert on Echo 2? (c) What force did sunlight exert on the satellite? (Hint: For the purposes of this calculation, you may consider the sphere to be a flat disk with a diameter of 41.0 m, directly facing the Sun.) (d) The mass of the Echo 2 balloon was 256 kg. What was the magnitude of its acceleration due to the pressure of sunlight?

(a) ________ W/m²
(b) ________ N/m²
(c) ________ N
(d) ________ m/s²

13.5 (a) The Sun repels the Earth by pushing on it with sunlight. What is the force exerted by sunlight pressure on the planet Earth? Treat the Earth as a flat disk facing the Sun. Its radius is 6.37×10⁶ m, and its albedo is 0.39. The intensity of sunlight at the Earth's orbit is 1380 W/m². (b) The Sun attracts the Earth by pulling on it with gravity. What is the force exerted by the Sun's gravity on the planet Earth? The mass of the Earth is 5.97×10²⁴ kg, the mass of the Sun is 1.99×10³⁰ kg, and the distance between the two bodies is 1.50×10¹¹ m.

(a) ________ N
(b) ________ N
13.6 Clouds of interstellar dust and gas coalesce into nodes, which further contract into protostars that eventually give birth to new stars. The mutual gravitational attraction of dust particles in the cloud accounts for some of this attraction, and it becomes the dominant factor as the cloud gathers into an opaque mass. During the early stages of contraction, however, the inward light pressure of the entire surrounding universe on the dust cloud is the dominant factor. How does this compressing effect vary with the size of the dust cloud, which is to say with the distances between its particles? In this problem you will answer this question by considering how isotropic light pressure provides an "attractive" force between two perfectly reflective dust particles.

(a) Imagine a three-dimensional $xyz$ coordinate system in interstellar space. Two spherical dust particles of fixed diameter lie on the $x$ axis at a variable distance $r$ from each other, one on either side of the origin. Consider the sum of the light from all the stars with positive $x$ coordinates that illuminates these particles. Which particle does this light repel with greater force? Explain your answer.

(b) The "positive $x$" particle casts a diffuse shadow on the "negative $x$" particle. The effective area of the shadow is proportional to the fraction of the hemispherical "positive $x$" sky that the "positive $x$" particle obscures, from the viewpoint of the "negative $x$" particle. What power of the distance $r$ between the particles is this obscured fraction proportional to?

(c) The "positive $x$" sky repels both particles, but it repels the "negative $x$" particle less than it does the "positive $x$" particle because of the shadow. The difference between these repulsive forces is equivalent to an attractive force between the particles in their own frame of reference. Let $I$ be the intensity of the net $x$ component of all the starlight from the "positive $x$" sky, and let $A$ be the effective area of the shadow that the "positive $x$" particle casts on the "negative $x$" particle. What is the magnitude $F$ of the resulting "attractive force" between the particles?

(d) Symmetrically, the "negative $x$" particle shadows the "positive $x$" particle from the "negative $x$" sky, doubling the light pressure "attraction" between the particles. Considering your answers to parts b and c, what kind of proportionality relates the total "attractive force" between the particles and the distance $r$ between them?

(a) ( ) The "positive $x$" particle  ( ) The "negative $x$" particle

(b) ( ) $r^2$  ( ) $r$  ( ) $1/r$  ( ) $1/r^2$

(c) ( ) $F = 2I/ac$  ( ) $F = I/c$  ( ) $F = 2I/ac$  ( ) $F = I/ac$

(d) ( ) Direct square
   (ii) Direct
   (iii) Inverse
   (iv) Inverse square

13.7 A comet is a huge "dirty snowball" that orbits the Sun in an extremely long, eccentric orbit that loops close around the Sun at one end and travels far out into space on the other. Each time it nears a close encounter with the Sun, it is heated so that copious amounts of dust and gas boil off its surface (resulting in a lifetime that is quite limited in astronomical terms). Dust particles that boil off the comet do not follow on in its orbit. Instead, they are blown away from the Sun by the pressure of sunlight. Assume that each dust particle is spherical, with radius $r$, albedo 0.37, and density $3.90 \times 10^3$ kg/m$^3$. What is the radius $r$ of a dust particle that is equally repelled by the Sun's light and attracted by its gravity?

$$ r = \text{m} $$

Section 18 - Polarization and intensity

18.1 A beam of unpolarized light of intensity 30.0 W/m$^2$ passes through a polarizing sheet. What is the electric field amplitude of the transmitted beam?

$$ E = \text{V/m} $$
This problem deals with combinations of ideal polarizing filters that have the net effect, first of polarizing incident light, and then of rotating the plane of polarization by 90°. In the text it was shown that two crossed polarizing filters (at 90° to each other) completely block initially unpolarized light that falls on the pair of filters. The first filter linearly polarizes the incoming light, resulting in an initially polarized intensity of $I_p$. Then the second crossed filter blocks all of the now-polarized light, and 0% intensity is transmitted. Somewhat surprisingly, adding a third filter between the crossed filters, so that each polarizer is rotated 45° with respect to the one just before it, does transmit some light. Again, the first filter linearly polarizes the incoming light, but the other filters cumulatively rotate this plane of polarization by 90° without blocking all of the light. In this case $I_{\text{final}} = 0.25I_p$, so 25.0% of the intensity emerging from the initial polarizer is transmitted through all three filters.

A combination of four filters, each rotated 30.0° clockwise with respect to the previous one, is exposed to light. This combination results in a 90° clockwise rotation of the plane of polarization that emerges from the first of the filters. What fraction of this initially polarized intensity $I_p$ is the final intensity of the polarized light that emerges after passing through the other filters? Express your answer in terms of a percentage of $I_p$.

% 

Before working this problem, read as introductory background the previous homework problem which dealt with combinations of two, three, and four polarizing filters that have the net effect of polarizing incident light, and then rotating the plane of polarization by 90°. A combination of 91 ideal polarizing filters, each rotated 1.00° clockwise with respect to the previous one, is exposed to unpolarized light. This combination results in a 90° clockwise rotation of the initial plane of polarization. The first filter linearly polarizes the incoming light. Call this initially polarized intensity $I_p$. What fraction of $I_p$ is the final intensity of the polarized light that emerges after passing through all the filters? Express your answer in terms of the percentage of the intensity transmitted. 

% 

Section 22 - Optically active substances

The organic chemical lactic acid occurs in two mirror image forms. D-lactic acid (which rotates polarized light clockwise) is produced by muscles as they work, and its excessive buildup is the fatigue poison that causes discomfort in overworked limbs. L-lactic acid (which rotates polarized light counterclockwise) is an ingredient of milk. A scientist wishes to establish an extremely accurate value for the specific rotation $\alpha_0$ of lactic acid. She obtains a sample of pure L-lactic acid from milk, and then dilutes it with optically inactive water to a concentration of 325.0 kg/m$^3$. She measures the amount of counterclockwise rotation in the plane of 589 nm polarized light from a sodium lamp that results after the light passes through a 10.00 m long tube filled with the solution, and finds that it is 81.37°. (She knows the approximate specific rotation of lactic acid, so she is sure that the rotation really is this amount, and not the same angle plus some multiple of 180°.) What is the specific rotation of lactic acid, to four significant figures?

$\alpha_0$ m$^2$/kg

Molecules of the organic terpene limonene occur naturally in mirror image forms, called D-limonene and L-limonene depending on whether they rotate polarized light clockwise (D) or counterclockwise (L). D-limonene is the essential oil that conveys the scent of orange, while L-limonene carries the scent of lemon. L-limonene is frequently used to add lemon scent to household products such as dish soaps and oven cleaners. The specific rotation of limonene is $\alpha_0 = 120$ °m$^2$/kg. L-limonene occurs naturally in lemon peels, but it is also a major constituent of a far cheaper byproduct of the timber industry, pine oil, where it occurs mixed with substantial amounts of D-limonene. Limonene refined from crushed pine needles is a variable mixture of both forms of the molecule, and there is no economical way to separate them. Perfume manufacturers wish to make sure that the expensive limonene they purchase for use in scents is pure L-limonene, uncontaminated by D-limonene. A scientist uses optically inactive alcohol to dilute a solution of limonene to a concentration of 125 kg/m$^3$. Testing it with a polarimeter, he finds that a sample of the solution 15.6 cm thick rotates polarized light 21.5° counterclockwise. What percent of the limonene is the desired form, L-limonene?

%