10.0 - Introduction

If you feel as though you spend your life spinning around in circles, you may be pleased to know that an entire branch of physics is dedicated to studying that kind of motion. This chapter is for you! More seriously, this chapter discusses motion that consists of rotation about a fixed axis. This is called pure rotational motion. There are many examples of pure rotational motion: a spinning Ferris wheel, a roulette wheel, or a music CD are three instances of this type of motion.

In this chapter, you will learn about rotational displacement, rotational velocity, and rotational acceleration: the fundamental elements of what is called rotational kinematics.

You will also learn how to relate these quantities using equations quite similar to those used in the study of linear motion.

The simulation on the right features the "Angular Surge," an amusement park ride you will be asked to operate in order to gain insight into rotational kinematics. The ride has a rotating arm with a "rocket" where passengers sit. You can move the rocket closer to or farther from the center by setting the distance in the simulation. You can also change the rocket's period, which is the amount of time it takes to complete one revolution.

By changing these parameters, you affect two values you see displayed in gauges: the rocket's angular velocity and its linear speed. The rocket's angular velocity is the change per second in the angle of the ride's arm, measured from its initial position. Its unit is radians per second. For instance, if the rocket completes one revolution in one second, its angular velocity is $2\pi$ radians (360°) per second.

This simulation has no specific goal for you to achieve, although you may notice that you can definitely have an impact on the passengers! What you should observe is this: How do changes in the period affect the angular velocity? The linear speed? And how does a change in the distance from the center (the radius of the rocket's motion) affect those values, if at all? Can you determine how to maximize the linear speed of the rocket?

To run the ride, you start the simulation, set the values mentioned above, and press GO. You can change the settings while the ride is in motion.

10.1 - Angular position

Angular position: The amount of rotation from a reference position, described with a positive or negative angle.

When an object such as a bicycle wheel rotates about its axis, it is useful to describe this motion using the concept of angular position. Instead of being specified with a linear coordinate such as $x$, as linear position is, angular position is stated as an angle.

In Concept 1, we use the location of a bicycle wheel's valve to illustrate angular position. The valve starts at the 3 o'clock position (on the positive $x$ axis), which is zero radians by convention. As the illustration shows, the wheel has rotated one-eighth of a turn, or $\pi/4$ radians (45°), in a counterclockwise direction away from the reference position. In other words, angular position is measured from the positive $x$ axis.

Note that this description of the wheel's position used radians, not degrees; this is because radians are typically used to describe angular position. The two lines we use to measure the angle radiate from the point about which the wheel rotates.

The axis of rotation is a line also used to describe an object's rotation. It passes through the wheel's center, since the wheel rotates about that point, and it is perpendicular to the wheel. The axis is assumed to be stationary, and the wheel is assumed to be rigid and to maintain a constant shape. Analyzing an object that changes shape as it rotates, such as a piece of soft clay, is beyond the scope of this textbook. We are concerned with the wheel's rotational motion here; its motion around a fixed axis. Its linear motion when moving along the ground is another topic.

As mentioned, angular position is typically measured with radians (rad) instead of degrees. The formula that defines the radian measure of an angle is shown in Equation 1. The angle in radians equals the arc length $s$ divided by the radius $r$. As you may recall, $2\pi$ radians equals one revolution around a circle, or 360°. One radian equals about 57.3°. To convert radians to degrees, multiply by the conversion factor 360°/$2\pi$. To convert degrees to radians, multiply by the reciprocal: $2\pi/360°$. The Greek letter $\theta$
(theta) is used to represent angular position.

The angular position of zero radians is defined to be at 3 o'clock, which is to say along a horizontal line pointing to the right. Let's now consider what happens when the wheel rotates a quarter turn **counterclockwise**, moving the valve from the 3 o'clock position to 12 o'clock. A quarter turn is \( \pi/2 \) rad (or 90°). The valve’s angular position when it moves a quarter turn counterclockwise is \( \pi/2 \) rad. By convention, angular position **increases** with counterclockwise motion.

The valve can be placed in the same angular position, \( \pi/2 \) rad, by rotating the wheel in the other direction, by rotating it **clockwise** three quarters of a turn. By convention, angular position **decreases** with clockwise motion, so this rotation would be described as an angular position of \(-3\pi/2\) rad.

An angular position can be greater than \( 2\pi \) rad. An angular position of \( 3\pi \) rad represents one and a half counterclockwise revolutions. The valve would be at 9 o'clock in that position.

\[ \theta = \frac{s}{r} \]

\( \theta \) = angle in radians  
\( s \) = arc length  
\( r \) = radius

**What is the arc length?**

\[ s = r\theta = (0.35 \text{ m})(\pi/3 \text{ rad}) \]

\[ s = 0.37 \text{ m} \]

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**Angular displacement:** Change in angular position.

In Concept 1 you see a pizza topped with a single mushroom (we are not going back to that pizzeria!). We use a mushroom to make the rotational motion of the pizza easier to see. As the pizza rotates, its angular position changes. This change in angular position is called angular displacement.

To calculate angular displacement, you subtract the initial angular position from the final position. For instance, the mushroom in the Equation illustration moves from \( \pi/2 \) rad to \( \pi \) rad, a displacement of \( \pi/2 \) rad. As you can see in this example, angular displacement in the counterclockwise direction is positive.

**Revolution** is a common term in the study of rotational motion. It means one complete rotational cycle, with the object starting and returning to the same position. One counterclockwise revolution equals \( 2\pi \) radians of angular displacement.

The angular displacement is the **total** angle “swept out” during rotational motion from an initial to a final position. If the pizza turns counterclockwise three complete revolutions, its angular displacement is \( 6\pi \) radians.

The definition of angular displacement resembles that of linear displacement. However, the discussion above points out a difference. A mushroom that makes a complete revolution has an angular displacement of \( 2\pi \) rad. On the other hand, its linear displacement equals zero, since it starts and stops at the same point.

**Angular displacement**

\[ \Delta \theta = \theta_f - \theta_i \]

\( \Delta \theta \) = angular displacement  
\( \theta_f \) = final angular position  
\( \theta_i \) = initial angular position

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At 12:10, the initial angular position of the minute hand is $\pi/6$. After 15 minutes have passed, what is the minute hand’s angular displacement?

$$\Delta \theta = \theta_f - \theta_i$$

$$\Delta \theta = \left( -\frac{\pi}{3} \text{ rad} \right) - \left( \frac{\pi}{2} \text{ rad} \right)$$

$$\Delta \theta = -\frac{\pi}{6} \text{ rad}$$

**Angular velocity:** Angular displacement per unit time.

In Concept 1, a ball attached to a string is shown moving counterclockwise around a circle. Every four seconds, it completes one revolution of the circle. Its angular velocity is the angular displacement $2\pi$ radians (one revolution) divided by four seconds, or $\pi/2$ rad/s. The Greek letter $\omega$ (omega) represents angular velocity.

As is the case with linear velocity, angular velocity can be discussed in terms of average and instantaneous velocity. Average angular velocity equals the total angular displacement divided by the elapsed time. This is shown in the first equation in Equation 1.

Instantaneous angular velocity refers to the angular velocity at a precise moment in time. It equals the limit of the average velocity as the increment of time approaches zero. This is shown in the second equation in Equation 1.

The sign of angular velocity follows that of angular displacement: positive for counterclockwise rotation and negative for clockwise rotation. The magnitude (absolute value) of angular velocity is **angular speed**.
Angular acceleration: The change in angular velocity per unit time.

By now, you might be experiencing a little déjà vu in this realm of angular motion. Angular velocity equals angular displacement per unit time, but if you drop the word “angular” you are stating that velocity equals displacement per unit time, an equation that should be familiar to you from your study of linear motion.

So it is with angular acceleration. Angular acceleration equals the change in angular velocity divided by the elapsed time. The toy train shown in Concept 1 is experiencing angular acceleration. This is reflected in the increasing separation between the images you see. Its angular velocity is becoming increasingly negative since it is moving in the clockwise direction. It is moving faster and faster in the negative angular direction.

Average angular acceleration equals the change in angular velocity divided by the elapsed time. The instantaneous angular acceleration equals the limit of this ratio as the increment of time approaches zero. These two equations are shown in Equation 1 to the right. The Greek letter $\alpha$ (alpha) is used to represent angular acceleration.

With rotational kinematics, we often pose problems in which the angular acceleration is constant; this helps to simplify the mathematics involved in solving problems. We made similar use of constant acceleration for the same reason in the linear motion chapter.
\[ \alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \]

\( \alpha \) = instantaneous angular acceleration  
\( \omega \) = angular velocity  
\( \Delta t \) = elapsed time  
Units: rad/s^2

The toy train starts from rest and reaches the angular velocity shown in 5.0 seconds. What is its average angular acceleration?

\[ \overline{\alpha} = \frac{\Delta \omega}{\Delta t} \]

\[ \overline{\alpha} = \frac{0.50 \text{ rad/s} - 0.00 \text{ rad/s}}{5.0 \text{ s}} \]

\[ \overline{\alpha} = 0.10 \text{ rad/s}^2 \]

10.5 - Sample problem: a clock

Over the course of 1.00 hour, what is (a) the angular displacement, (b) the angular velocity and (c) the angular acceleration of the minute hand?

Think about the movement of the minute hand over the course of an hour. Be sure to consider the direction!

**Variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Example Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>elapsed time</td>
<td>( \Delta t = 1.00 \text{ h} )</td>
</tr>
<tr>
<td>angular displacement</td>
<td>( \Delta \theta )</td>
</tr>
<tr>
<td>angular velocity</td>
<td>( \omega )</td>
</tr>
<tr>
<td>angular acceleration</td>
<td>( \alpha )</td>
</tr>
</tbody>
</table>

**What is the strategy?**

1. Calculate the angular displacement.
2. Convert the elapsed time to seconds.
3. Use the angular displacement and time to determine the angular velocity and angular acceleration.

**Physics principles and equations**

- Definition of angular velocity
\[ \omega = \frac{\Delta \theta}{\Delta t} \]

Definition of angular acceleration

\[ \alpha = \frac{\Delta \omega}{\Delta t} \]

Step-by-step solution

We start by calculating the angular displacement of the minute hand over 1.00 hour. We then calculate the angular velocity.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \Delta \theta = -2\pi \text{ rad} )</td>
</tr>
<tr>
<td>2.</td>
<td>( \Delta t = 1.00 \text{ h} \left( \frac{3600 \text{ s}}{1.00 \text{ h}} \right) = 3600 \text{ s} )</td>
</tr>
<tr>
<td>3.</td>
<td>( \omega = \frac{\Delta \theta}{\Delta t} )</td>
</tr>
<tr>
<td>4.</td>
<td>( \omega = \frac{-2\pi \text{ rad}}{3600 \text{ s}} )</td>
</tr>
<tr>
<td>5.</td>
<td>( \omega = -1.75 \times 10^{-3} \text{ rad/s} )</td>
</tr>
</tbody>
</table>

The angular displacement is calculated in step 1, and the angular velocity in step 5. Since the angular velocity is constant, the angular acceleration is zero.

### 10.6 - Interactive checkpoint: a potter’s wheel

At a particular instant, a potter’s wheel rotates clockwise at 12.0 rad/s; 2.50 seconds later, it rotates at 8.50 rad/s clockwise. Find its average angular acceleration during the elapsed time.

**Answer:**

\[ \alpha = \text{ rad/s}^2 \]

### 10.7 - Equations for rotational motion with constant acceleration

\[ \omega_f = \omega_i + \alpha t \]

\[ \Delta \theta = \omega_i t + \frac{1}{2} \alpha t^2 \]

\[ \omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta \]

\[ \Delta \theta = \frac{1}{2} (\omega_i + \omega_f) t \]

- \( \Delta \theta = \) angular displacement
- \( \omega = \) angular velocity
- \( \alpha = \) angular acceleration
- \( t = \) elapsed time

Above, you see a table of equations for rotational motion. These equations resemble those for linear motion, except that they now are defined.
for angular displacement, angular velocity and angular acceleration instead of linear displacement, velocity and acceleration. As with the linear motion equations, these equations hold true when there is constant acceleration. We also show these equations below along with their linear counterparts.

To apply the equations in physics problems, the first step is to identify the known values and which values are being asked for. Sketching a diagram of the situation may help you with this.

The next step is to find an equation that includes both the known and the unknown (asked-for) values. Your goal is to find an equation, if possible, that has only one unknown value: the one you want to find.

When applying the rotational equations, remember that positive displacement and velocity represent counterclockwise motion, and negative displacement and velocity indicate clockwise motion.

Let’s now work an example problem. Imagine you have just turned on the blender shown on the right. You let it run for 5.0 seconds. During this time period its blade has a constant angular acceleration of 44 radians per second squared. What is the angular displacement of the blade during this time?

This problem implicitly tells you that the initial angular velocity is zero, since the blender has just been turned on. The second equation above includes time, initial angular velocity and acceleration. It also contains the value you seek to calculate: the angular displacement. This makes it the right equation to use. It does not include the value for final angular velocity, which is fine because you are not told that value, nor are you asked to calculate it.

The details of the calculation appear on the right. The angular displacement is 550 radians. Because the value is positive, the motion is counterclockwise.

Here is a table of the rotational motion variables and the equations that relate them, along with their linear counterparts.

<table>
<thead>
<tr>
<th>linear</th>
<th>rotational</th>
</tr>
</thead>
<tbody>
<tr>
<td>position</td>
<td>$x$</td>
</tr>
<tr>
<td>displacement</td>
<td>$\Delta x$</td>
</tr>
<tr>
<td>velocity</td>
<td>$v = \Delta x / \Delta t$</td>
</tr>
<tr>
<td>acceleration</td>
<td>$a = \Delta v / \Delta t$</td>
</tr>
<tr>
<td>$v_f = v_i + at$</td>
<td>$\omega_f = \omega_i + \alpha t$</td>
</tr>
<tr>
<td>$\Delta x = v_i t + \frac{1}{2} a t^2$</td>
<td>$\Delta \theta = \omega_i t + \frac{1}{2} \alpha t^2$</td>
</tr>
<tr>
<td>$v_f^2 = v_i^2 + 2a\Delta x$</td>
<td>$\omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta$</td>
</tr>
<tr>
<td>$\Delta x = \frac{1}{2}(v_i + v_f)t$</td>
<td>$\Delta \theta = \frac{1}{2}(\omega_i + \omega_f)t$</td>
</tr>
</tbody>
</table>

**Examples**

### Sample problem: a carousel

The carousel accelerates from rest for two revolutions at a constant angular acceleration of 0.11 rad/s$^2$. What is its final angular velocity?

<table>
<thead>
<tr>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial angular velocity</td>
</tr>
<tr>
<td>final angular velocity</td>
</tr>
<tr>
<td>angular acceleration</td>
</tr>
<tr>
<td>angular displacement</td>
</tr>
</tbody>
</table>
What is the strategy?

1. Identify the known and unknown values and choose an equation.
2. Substitute the known values and solve the equation for the final angular velocity.

Physics principles and equations

For the known and unknown values in this problem, the appropriate equation is

\[ \omega_f^2 = \omega_i^2 + 2a\Delta\theta \]

This includes the values stated above and the value we are asked to determine.

Step-by-step solution

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \omega_f^2 = \omega_i^2 + 2a\Delta\theta )</td>
<td>rotational motion equation</td>
</tr>
<tr>
<td>2. ( \omega_f^2 = (10 \text{ rad/s})^2 + 2(0.2 \text{ rad/s}^2)(2\pi \text{ rad}) )</td>
<td>enter values</td>
</tr>
<tr>
<td>3. ( \omega_f^2 = 2.76 \text{ rad}^2/\text{s}^2 )</td>
<td>evaluate</td>
</tr>
<tr>
<td>4. ( \omega_f = 1.7 \text{ rad/s} )</td>
<td>square root</td>
</tr>
</tbody>
</table>

10.9 - Interactive checkpoint: roulette

A roulette wheel is spun counterclockwise with an initial angular velocity of 15.0 rad/s. If it slows down with a constant angular acceleration of -0.200 rad/s² until it stops, what is the total angular displacement of the wheel?

Answer:

\[ \Delta\theta = \text{ rad} \]

10.10 - Interactive problem: launch the rocket

Lucky you! You just landed a summer job operating the Angular Surge ride at a local amusement park. As shown on the right, the ride has a rotating arm with a rocket for transporting riders. The rocket is fixed to the end of the arm, and you control the angular acceleration of the ride for the first revolution. Your goal is to set this constant acceleration so that, after exactly one revolution, the ride has an angular velocity of 1.64 rad/s. If you set this value correctly, the rocket will take off. It must be moving at precisely this angular velocity to have the correct amount of energy to safely launch.

The angular acceleration you specify will be applied for exactly one revolution and then the rocket arm will maintain its velocity. Calculate the angular acceleration, enter the value and press GO to see the results. If you do not succeed in launching the rocket, press RESET to try again.

If you have difficulty with this problem, review the rotational motion equations and make sure you have chosen the appropriate one to solve the problem.
**Tangential velocity**: The instantaneous linear velocity of a point on a rotating object.

Concepts such as angular displacement and angular velocity are useful tools for analyzing rotational motion. However, they do not provide the complete picture. Consider the salt and pepper shakers rotating on the lazy Susan shown to the right. The containers have the same angular velocity because they are on the same rotating surface and complete a revolution in the same amount of time.

However, at any instant, they have different linear speeds and velocities. Why? They are located at different distances from the axis of rotation (the center of the lazy Susan), which means they move along circular paths with different radii. The circular path of the outer shaker is longer, so it moves farther than the inner one in the same amount of time. At any instant, its linear speed is greater. Because the direction of motion of an object moving in a circle is always tangent to the circle, the object’s linear velocity is called its tangential velocity.

To reinforce the distinction between linear and angular velocity, consider what happens if you decide to run around a track. Let’s say you are asked to run one lap around a circular track in one minute flat. Your angular velocity is \( 2 \pi \) radians per minute.

Could you do this if the track had a radius of 10 meters? The answer is yes. The circumference of that track is \( 2 \pi r \), which equals approximately 63 meters. Your pace would be that distance divided by 60 seconds, which works out to an easy stroll of about 1.05 m/s (3.78 km/h).

What if the track had a radius of 100 meters? In this case, the one-minute accomplishment would require the speed of a world-class sprinter capable of averaging more than 10 m/s. (If the math ran right past you, note that we are again multiplying the radius by \( 2 \pi \) to calculate the circumference and dividing by 60 seconds to calculate the tangential velocity.) Even though the angular velocity is the same in both cases, \( 2 \pi \) radians per minute, the tangential speed changes with the radius.

As you see in Equation 1, tangential speed equals the product of the distance to the axis of rotation, \( r \), and the angular velocity, \( \omega \). The units for tangential velocity are meters per second. The direction of the velocity is always tangent to the path of the object.

Confirming the direction of tangential velocity can be accomplished using an easy home experiment. Let’s say you put a dish on a lazy Susan and then spin the lazy Susan faster and faster. Initially, the dish moves in a circle, constrained by static friction. At some point, though, it will fly off. The dish will always depart in a straight line, tangent to the circle at its point of departure.

The tangential speed equation can also be used to restate the equation for centripetal acceleration in terms of angular velocity. Centripetal acceleration equals \( v^2/r \). Since \( v = ro \), centripetal acceleration also equals \( \omega^2 r \).

We derive the equation for tangential speed using the diagram below.

\[
\Delta s = r \Delta \theta
\]

To understand the derivation, you must recall that the arc length \( \Delta s \) (the distance along the circular path) equals the angular displacement \( \Delta \theta \) in radians times the radius \( r \). Also recall that the instantaneous speed \( v \) equals the displacement divided by the elapsed time for a very small increment of time.
Combining these two facts, and the definition of angular velocity, yields the equation for tangential speed.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $v_T = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$</td>
<td>definition of instantaneous velocity</td>
</tr>
<tr>
<td>2. $\Delta s = r \Delta \theta$</td>
<td>definition of radian measure</td>
</tr>
<tr>
<td>3. $v_T = \lim_{\Delta t \to 0} \frac{r \Delta \theta}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$</td>
<td>substitute equation 2 into equation 1</td>
</tr>
<tr>
<td>4. $v_T = r \omega$</td>
<td>definition of angular velocity</td>
</tr>
</tbody>
</table>

10.12 - Tangential acceleration

**Tangential acceleration**: A vector tangent to the circular path whose magnitude is the rate of change of tangential speed.

As discussed earlier, an object moving in a circle at a constant speed is accelerating because its direction is constantly changing. This is called centripetal acceleration.

Now consider the mushroom on the pizza to the right. Let’s say the pizza has a positive angular acceleration. Since it is rotating faster and faster, its angular velocity is increasing. Since tangential speed is the product of the radius and the angular velocity, the magnitude of its tangential velocity is also increasing.

The magnitude of the tangential acceleration vector equals the rate of change of tangential speed. The tangential acceleration vector is always parallel to the linear velocity vector. When the object is speeding up, it points in the same direction as the tangential velocity vector; when the object is slowing down, tangential acceleration points in the opposite direction.

Since the centripetal acceleration vector always points toward the center, the centripetal and tangential acceleration vectors are perpendicular to each other. An object’s overall acceleration is the sum of the two vectors. To put it another way: The centripetal and tangential acceleration are perpendicular components of the object’s overall acceleration.

Like tangential velocity, tangential acceleration increases with the distance from the axis of rotation. Consider again the pizza and its toppings in Concept 1. Imagine that the pizza started stationary and it now has positive angular acceleration. Since tangential velocity is proportional to radius, at any moment in time the mushroom near the outer edge of the pizza has greater tangential velocity than the piece of pepperoni closer to the center. Since the mushroom’s change in tangential velocity is greater, it must have accelerated at a greater rate.

Tangential acceleration can be calculated as the product of the radius and the angular acceleration. This relationship is stated in Equation 1. The units for tangential acceleration are meters per second squared, the same as for linear acceleration. Note that it only makes sense to calculate the tangential acceleration for an object (or really a point) on the pizza. You cannot speak of the tangential acceleration of the entire pizza because it includes points that are at different distances from its center and have different rates of tangential acceleration.

Because it is easy to confuse angular and linear motion, we will now review a few fundamental relationships.

An object rotating at a constant angular velocity has zero angular acceleration and zero tangential acceleration. An example of this is a car driving around a circular track at a constant speed, perhaps at 100 km/hr. This means the car completes a lap at a constant rate, so its angular velocity is constant. A constant angular velocity means zero angular acceleration. Since the angular acceleration is zero, so is the tangential acceleration.

In contrast, the car’s linear (or tangential) velocity is changing since it changes direction as it moves along the circular path. This accounts for the car’s centripetal acceleration, which equals its speed squared divided by the radius of the track. The direction of centripetal acceleration is
always toward the center of the circle.

Now imagine that the car speeds up as it circles the track. It now completes a lap more quickly, so its angular velocity is increasing, which means it has positive angular acceleration (when it is moving counterclockwise, it is negative in the other direction). The car now has tangential acceleration (its linear speed is changing), and this can be calculated by multiplying its angular acceleration by the track’s radius.

The equation for tangential acceleration is derived below from the equations for tangential velocity and angular acceleration. We begin with the basic definition of linear acceleration and substitute the tangential velocity equation. The result is an expression which contains the definition of angular acceleration. We replace this expression with $a$, angular acceleration, which yields the equation we desire.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $a_T = \lim_{\Delta t \to 0} \frac{\Delta v_T}{\Delta t}$</td>
<td>definition of linear acceleration</td>
</tr>
<tr>
<td>2. $\Delta v_T = r \Delta \omega$</td>
<td>tangential velocity equation</td>
</tr>
<tr>
<td>3. $a_T = \lim_{\Delta t \to 0} \frac{r \Delta \omega}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$</td>
<td>substitute equation 2 into equation 1</td>
</tr>
<tr>
<td>4. $a_T = ra$</td>
<td>definition of angular acceleration</td>
</tr>
</tbody>
</table>

What is the tangential acceleration of the mushroom slice at this instant?

Answer:

$$a_T = r \frac{\pi}{10} \text{rad/s}^2(0.15 \text{ m}) = 0.047 \text{ m/s}^2$$

pointing down

10.13 - Interactive checkpoint: a marching band

The performers in a marching band move in straight rows, maintaining constant side-to-side spacing between them. Each row sweeps 90° through a circular arc when the band turns a corner. The radii of the paths followed by the marchers at the inner and outer ends of a row are 1.50 m and 7.50 m. If the innermost marcher in a row moves at 0.350 m/s, what is the speed of the outermost marcher?

Answer:

$$v_{out} = \boxed{m/s}$$

10.14 - Tangential and centripetal acceleration

In Concept 1, a toy train is shown going around a circular track at steadily increasing speed. How can we calculate its overall acceleration at any moment?

The train has both centripetal and tangential acceleration. The overall acceleration can be broken into these two components.

The acceleration perpendicular to the direction of motion, directed toward the center of the circle, is the centripetal acceleration. Its magnitude at any instant is calculated using the equation for centripetal acceleration from a previous chapter: speed squared divided by the radius.

The acceleration parallel to the velocity vector is the tangential acceleration, which is perpendicular to the centripetal acceleration. Since the train is increasing in speed, it has non-zero tangential acceleration. (This is not uniform circular motion.)

The overall acceleration equals the vector sum of the centripetal and tangential accelerations. The two vectors are perpendicular, so they form two legs of a right triangle. The Pythagorean theorem can be used to calculate the magnitude of the overall acceleration, as the first formula in Equation 1 shows. The direction of the overall acceleration, measured from the centripetal acceleration vector (or the radius line), can be calculated using trigonometry. You see that formula in Equation 1 as well.
Combining centripetal and tangential acceleration

Centripetal acceleration
- Points toward center of circular path
Tangential acceleration
- Reflects speeding or slowing
Overall acceleration is the vector sum

\[ \mathbf{a} = \mathbf{a}_c + \mathbf{a}_T \]

\[ a = \sqrt{a_c^2 + a_T^2} \]
\[ \theta = \arctan\left( \frac{a_T}{a_c} \right) \]

\( a \) = overall acceleration,
\( a_c \) = centripetal acceleration,
\( a_T \) = tangential acceleration,
\( \theta \) = angle of \( a \) relative to \( a_c \)

What are the magnitude and direction of the overall acceleration?

\[ a = \sqrt{a_c^2 + a_T^2} \]
\[ a = \sqrt{(0.85 \text{ m/s}^2)^2 + (0.42 \text{ m/s}^2)^2} \]
\[ a = 0.95 \text{ m/s}^2 \]
\[ \theta = \arctan\left( \frac{a_T}{a_c} \right) \]
Although we have not stressed this fact, angular velocity and angular acceleration are both vectors. In this section, we discuss the direction in which they point, using the right-hand rule to determine their direction. To apply this rule to angular velocity, curl your right hand around the axis of rotation, wrapping your fingers in the direction of the motion. This is illustrated to the right, where the hand wraps around the axis that passes through the center of the record. Your thumb then points in the direction of the angular velocity vector, which lies along the axis of rotation.

The direction of the angular acceleration vector depends on whether the object in question is speeding up or slowing down. When an object speeds up, the angular acceleration vector points in the same direction as the angular velocity vector, reflecting the change in the velocity vector. When an object slows down, the angular acceleration vector points in the direction opposite to the angular velocity vector, again reflecting the change in the angular velocity vector.

You may have noticed that we have not mentioned angular displacement. This is because it is not treated as a vector. Understanding why this is so requires a discussion of rotational motion outside the scope of this textbook.

Angular velocity vector
Along axis of rotation
Magnitude proportional to angular speed
Direction determined by right-hand rule

Angular acceleration vector
Speeding up: same direction as angular velocity vector
Slowing down: opposite direction

The motorcycle rider speeds up as she starts her ride. What is the direction of the angular velocity vector? The angular acceleration vector?
Angular velocity vector: up
Angular acceleration vector: up

You are again operating the Angular Surge ride at a local amusement park. The ride begins with the arm in the launch position for the rocket. The motor starts the ride by providing a constant positive angular acceleration for the first 11.6 seconds.

The ride has a rocket on a rotating arm, and you can control the arm’s angular acceleration. You can also control the distance of the rocket from the axis of rotation.

Your goal is to set both these values so that 11.6 seconds after startup, the rocket has completed one or more complete revolutions and has a tangential velocity of 13.0 m/s. If you do this correctly, the rocket will blast off. You can position the rocket from four to 10 meters from the
The rocket can complete one revolution, or multiple complete revolutions, as long as it returns to its initial position in 11.6 seconds. Here is one way to solve the problem: Start by calculating the angular acceleration that would be needed to complete one revolution in 11.6 seconds, to the nearest hundredth rad/s², and enter it in the space provided. From this, you can calculate the final angular velocity of the rocket. Can you set the radius of the rocket so that its tangential velocity is 13.0 m/s? If not, try again, using two complete revolutions.

When you have determined the angular acceleration you want to use and the radius required, set these values, then press GO. If you are correct, at 11.6 seconds the rocket will take off.

If you have difficulty with this problem, review the sections on the rotational motion equations and tangential velocity, and remember that more than one revolution may be necessary.

10.17 - Gotchas

A potter's wheel rotates. A location farther from the axis will have a greater angular velocity than one closer to the axis. Wrong. They all have the same angular displacement over time, which means they have the same angular velocity, as well. In contrast, they do have different linear (tangential) velocities.

A point on a wheel rotates from 12 o'clock to 3 o'clock, so its angular displacement is 90 degrees, correct? No. This would be one definite error and one "units police" error. The displacement is negative because clockwise motion is negative. And, using radians is preferable and sometimes essential in the study of angular motion, so the angular displacement should be stated as −\(\pi/2\) radians.

10.18 - Summary

Rotational kinematics applies many of the ideas of linear motion to rotational motion.

Angular position is described by an angle \(\theta\), measured from the positive x axis. Radians are the typical units.

Angular displacement is a change \(\Delta\theta\) in angular position. By convention, the counterclockwise direction is positive.

Angular velocity is the angular displacement per unit time. It is represented by \(\omega\) and has units of radians per second.

Angular acceleration is the change in angular velocity per unit time. It is represented by \(\alpha\) and has units of radians per second squared.

As with linear motion, physicists define instantaneous and average angular velocity and angular acceleration. Instantaneous and average are defined in ways analogous to those used in the study of linear motion.

The equations for rotational motion are the same as those for linear motion except that displacement, velocity and constant acceleration are replaced by angular displacement, angular velocity and angular acceleration. For the equations to hold true, the angular acceleration must be constant.

The linear velocity of a point on a rotating object is called its tangential velocity, because it is always directed tangent to its circular path. Any two points on a rigid rotating object have the same angular velocity, but do not have the same tangential velocity unless they are the same distance from the rotational axis. Tangential speed increases as the distance from the axis of rotation increases.

Tangential acceleration is the change in tangential speed per unit time. Its magnitude increases as the radius increases. Its direction is the same as the tangential velocity if the object is speeding up, and in the opposite direction as the velocity if it is slowing down.

You have now studied three types of acceleration relating to rotational motion. Centripetal acceleration is due to the change in direction of an object in circular motion. Tangential acceleration is the linear acceleration due to a change in angular velocity. Angular acceleration is the rate of change in angular velocity.

You can add the tangential and centripetal acceleration vectors of an object to determine the total linear acceleration of an object in rotational or circular motion.
Conceptual Problems

C.1 Is it possible for a rotating object to have increasing angular speed and negative angular acceleration? Explain your answer.

☐ Yes  ☐ No

C.2 Order these three cities from smallest to largest tangential velocity due to the rotation of the Earth: Washington, DC, USA; Havana, Cuba; Ottawa, Canada.

smallest:  
   i. Havana
   ii. Ottawa
   iii. Washington

middle:  
   i. Havana
   ii. Ottawa
   iii. Washington

largest:  
   i. Havana
   ii. Ottawa
   iii. Washington

C.3 Which of the following rotational quantities are the same for all points on a rotating disk? Check all that apply, and explain your selections.

☐ Angular velocity
☐ Tangential velocity
☐ Angular acceleration
☐ Tangential acceleration
☐ Centripetal acceleration

C.4 Does the angular velocity vector of the Earth point north or south along its axis of rotation?

☐ North  ☐ South

Section Problems

Section 0 - Introduction

0.1 Use the simulation in the interactive problem in this section to answer the following questions. (a) If you increase the period, will the angular velocity increase, decrease or stay the same? (b) If you increase the period, will the linear speed increase, decrease or stay the same? (c) If you increase the distance from the center, will the angular velocity increase, decrease or stay the same? (d) If you increase the distance from the center, will the linear speed increase, decrease or stay the same?

(a)  
   i. Increase
   ii. Stay the same
   iii. Decrease

(b)  
   i. Increase
   ii. Stay the same
   iii. Decrease

(c)  
   i. Increase
   ii. Stay the same
   iii. Decrease

(d)  
   i. Increase
   ii. Stay the same
   iii. Decrease

0.2 Using the simulation in the interactive problem in this section and referring to your answers to the previous problem, what is the best way to maximize the linear speed of the rocket? Test your answer using the simulation.

   i. Maximize both the period and the distance from the center
   ii. Maximize the period and minimize the distance from the center
   iii. Minimize both the period and the distance from the center
   iv. Minimize the period and maximize the distance from the center
Section 1 - Angular position

1.1 Two cars are traveling around a circular track. The angle between them, from the center of the circle, is 55° and the track has a radius of 50 m. How far apart are the two cars, as measured around the curve of the track?

\[ \text{m} \]

1.2 Glenn starts his day by walking around a circular track with radius 48 m for 15 minutes. First he walks in a counterclockwise direction for 1000 meters, then he walks clockwise until the 15 minutes are up. This morning, his clockwise walk is 880 meters long. When he ends his walk, what is his angular position with respect to where he starts?

\[ \text{rad} \]

Section 2 - Angular displacement

2.1 A dancer completes 2.2 revolutions in a pirouette. What is her angular displacement?

\[ \text{rad} \]

2.2 What is the angular displacement in radians that the minute hand of a watch moves through from 3:15 A.M. to 7:30 P.M. the same day? Express your answer to the nearest whole radian.

\[ \text{rad} \]

2.3 What is the angular displacement in radians of the Earth around the Sun in one hour? Assume the orbit is circular and takes exactly 365 days in a counterclockwise direction, as viewed from above the North Pole.

\[ \text{rad} \]

2.4 The radius of the tires on your car is 0.33 m. You drive 1600 m in a straight line. What is the angular displacement of a point on the outer rim of a tire, around the center of the tire, during this trip? Assume the tire rotates in the counterclockwise direction.

\[ \text{rad} \]

2.5 A heavy vault door is shut. The angular position of the door from \( t = 0 \) to the time the door is shut is given by \( \theta(t) = 0.125t^2 \), where \( \theta \) is in radians. (a) The door is completely shut at \( \theta = \frac{\pi}{2} \) radians. At what time does this occur? (b) What is the angular displacement of the door between \( t = 0.52 \) s and \( t = 1.67 \) s? (c) What is the door's average angular velocity between \( t = 1.50 \) s and \( t = 2.50 \) s?

(a) \[ \text{s} \]
(b) \[ \text{rad} \]
(c) \[ \text{rad/s} \]

Section 3 - Angular velocity

3.1 A hamster runs in its wheel for 2.7 hours every night. If the wheel has a 6.8 cm radius and its average angular velocity is 3.0 radians per second, how far does the hamster run in one night?

\[ \text{m} \]

3.2 An LP record rotates at 33 1/3 rpm (revolutions per minute) and is 12.0 inches in diameter. What is the angular velocity in rad/s for a fly sitting on the outer edge of an LP rotating in a clockwise direction?

\[ \text{rad/s} \]

3.3 What is the average angular velocity of the Earth around the sun? Assume a circular counterclockwise orbit, and 365 days in a year.

\[ \text{rad/s} \]

3.4 A car starts a race on a circular track and completes the first three laps in a counterclockwise direction in 618 seconds, finishing with an angular velocity of 0.0103 rad/s. What is the car's average angular velocity for the first three laps?

\[ \text{rad/s} \]

3.5 Your bicycle tires have a radius of 0.33 m. It takes you 850 seconds to ride 14 times counterclockwise around a circular track of radius 73 m at constant speed. (a) What is the angular velocity of the bicycle around the track? (b) What is the magnitude of the angular velocity of a tire around its axis? (That is, don't worry about whether the tire's rotation is clockwise or counterclockwise.)

(a) \[ \text{rad/s} \]
(b) \[ \text{rad/s} \]
Section 4 - Angular acceleration

4.1 The blades of a fan rotate clockwise at \(-225 \text{ rad/s}\) at medium speed, and \(-355 \text{ rad/s}\) at high speed. If it takes 4.65 seconds to get from medium to high speed, what is the average angular acceleration of the fan blades during this time?

\[
\text{rad/s}^2
\]

4.2 The blades of a kitchen blender rotate counterclockwise at \(2.2 \times 10^4 \text{ rpm}\) (revolutions per minute) at top speed. It takes the blender 2.1 seconds to reach this top speed after being turned on. What is the average angular acceleration of the blades?

\[
\text{rad/s}^2
\]

4.3 A potter's wheel is rotating at \(3.2 \text{ rad/s}\) in a counterclockwise direction when the potter turns it off and lets it slow to a stop. This takes 26 seconds. What is the average angular acceleration of the wheel during this time?

\[
\text{rad/s}^2
\]

Section 7 - Equations for rotational motion with constant acceleration

7.1 A merry-go-round is at rest before a child pushes it so that it rotates with a constant angular acceleration for 27.0 s. When the child stops pushing, the merry-go-round is rotating at \(1.20 \text{ rad/s}\). How many revolutions did the child make around the merry-go-round while he was pushing it?

\[
\text{rev}
\]

7.2 A CD is rotating counterclockwise at \(31 \text{ rad/s}\). What angular acceleration will bring it to a stop in 28 rad?

\[
\text{rad/s}^2
\]

7.3 A motorcycle rider starts from rest and goes \(3.75 \text{ rad}\) counterclockwise around a circular track in 12.3 seconds. Assuming she accelerates at a constant rate, what is her angular acceleration?

\[
\text{rad/s}^2
\]

7.4 A rotating water pump works by taking water in at one side of a rotating wheel, and expelling it from the other side. If a pump with a radius of \(0.120 \text{ m}\) starts from rest and accelerates at \(30.5 \text{ rad/s}^2\), how fast will the water be traveling when it leaves the pump after it has been accelerating for 9.00 seconds?

\[
\text{m/s}
\]

7.5 A cyclist starts from rest and rides in a straight line, increasing speed so that her wheels have a constant angular acceleration of \(2.0 \text{ rad/s}^2\) around their axles. She accelerates until her wheels are rotating at \(8.0 \text{ rad/s}\). If the radius of a tire is 0.29 meters, how far has the cyclist traveled?

\[
\text{m}
\]

7.6 Many telescopes are housed in observatory domes, and a slit in the dome is opened to allow the telescope to see the sky. The dome has to be rotated so that the slit lines up with the telescope. In one observatory, there is a motor which causes the dome to rotate counterclockwise at \(0.0800 \text{ rad/s}\). When the motor is shut off, the dome continues to rotate, but with an angular acceleration of \(-0.0400 \text{ rad/s}^2\) until the angular velocity is zero. The dome is currently rotating, and the telescope is pointed at an angle of positive 1.30 radians from north. At what angle from north should the slit be located when you shut off the motor, so that the slit lines up exactly with the telescope when the dome stops moving?

\[
\text{rad}
\]

7.7 An athlete jogs at a constant angular velocity of \(0.150 \text{ rad/s}\) around a circular track. At time \(t = 0\), she passes a stationary runner, who immediately starts chasing her with a constant angular acceleration of \(0.225 \text{ rad/s}^2\). At what time will the second runner have caught up to the first runner?

\[
\text{s}
\]

7.8 A coin is rolled without slipping on a table top in a straight line. It starts rolling at \(3.4 \text{ rad/s}\) and slows down at a constant angular acceleration. It is rolling at \(1.2 \text{ rad/s}\) when it falls off the table edge. If the radius of the coin is \(0.011 \text{ m}\), and the edge of the table is \(1.6 \text{ m}\) from where the coin started, for how much time did the coin roll?

\[
\text{s}
\]

7.9 The platter of a modern hard disk drive spins at \(7.20 \times 10^3 \text{ rpm}\) (revolutions per minute). (a) How much time, in seconds, does it take for the disk to make a complete revolution? (b) Starting from rest, suppose the disk reaches full speed in 5.00 seconds. What is the average angular acceleration of the disk in radians per second? (c) Assuming constant angular acceleration, how many revolutions has the hard disk turned while spinning up to its final angular velocity?

(a) \[
\text{s}
\]

(b) \[
\text{rad/s}^2
\]

(c) \[
\text{rev}
\]
7.10 Two cars race around a circular track. Car A accelerates at 0.340 rad/s² around the track, and car B at 0.270 rad/s². They start at the same place on the track and car A lets the slower-to-accelerate car B start first. Car B starts at time \( t = 0 \). When car A starts, car B has an angular velocity of 1.40 rad/s. At what time does car A catch up to car B?

\[ \text{seconds} \]

**Section 10 - Interactive problem: launch the rocket**

10.1 Use the information given in the interactive problem in this section to answer the following question. What is the angular acceleration required for the rocket to reach the desired angular velocity after one revolution? Test your answer using the simulation.

\[ \text{rad/s}^2 \]

**Section 11 - Tangential velocity**

11.1 How might a magician make the Statue of Liberty disappear? Imagine that you are sitting with some spectators on a circular platform that, unknown to all of you, can rotate very slowly. It is evening, and you can see the Statue of Liberty a short distance away between two tall brightly lit columns at the rim of the platform. A large curtain can be drawn between the columns to temporarily hide the statue. The magician closes the curtain, then rotates the platform through an angle of just 0.170 radians so the statue is hidden behind one of the columns when the curtain is opened. (a) If the platform rotation takes 24.0 seconds, what is the average angular speed required? (b) You are sitting 4.00 m from the center of rotation while the platform is rotating. What is the centripetal acceleration required to move you along the circular arc? (c) Calculate the centripetal acceleration as a fraction of \( g \). You could be unaware of the rotation, especially if you were distracted.

(a) \[ \text{rad/s} \]

(b) \[ \text{m/s}^2 \]

(c) \[ \text{g} \]

11.2 An old-fashioned LP record rotates at 33 1/3 rpm (revolutions per minute) and is 12 inches in diameter. A "single" rotates at 45 rpm and is 7.0 inches in diameter. If a fly sits on the edge of an LP and then on the edge of a single, on which will the fly experience the greater tangential speed?

- On the LP
- On the 45

11.3 A computer hard drive disk with a diameter of 3.5 inches rotates at 7200 rpm. The "read head" is positioned exactly halfway from the axis of rotation to the outer edge of the disk. What is the tangential speed in m/s of a point on the disk under the read head?

\[ \text{m/s} \]

11.4 A radio-controlled toy car has a top speed of 7.90 m/s. You tether it to a pole with a rigid horizontal rod and let it drive in a circle at top speed. (a) If the rod is 1.80 m long, how long does it take the car to complete one revolution? (b) What is the angular velocity?

(a) \[ \text{s} \]

(b) \[ \text{rad/s} \]

11.5 You accelerate your car from rest at a constant rate down a straight road, and reach 22.0 m/s in 11.1 s. The tires on your car have radius 0.320 m. Assuming the tires rotate in a counterclockwise direction, what is the angular acceleration of the tires?

\[ \text{rad/s}^2 \]

11.6 When a compact disk is played, the angular velocity varies so that the tangential speed of the area being read by the player is constant. If the angular velocity of the CD when the player is reading at a distance of 3.00 cm from the center is 3.51 revolutions per second, what is the angular velocity when the player is reading at a distance of 4.00 cm from the center?

\[ \text{rev/s} \]

**Section 12 - Tangential acceleration**

12.1 A whirling device is launched spinning counterclockwise at 35 rad/s. It slows down with a constant angular acceleration and stops after 16 seconds. If the radius of the device is 0.038 m, what is the magnitude of the tangential acceleration of a point on the edge of the device?

\[ \text{m/s}^2 \]
12.2 Two go-karts race around a course that has concentric circular tracks. The radius of the inner track is 15.0 m, and the radius of the outer track is 19.0 m. The go-karts start from rest at the same angular position and time, and move at the same constant angular acceleration. The race ends in a tie after one complete lap, which takes 21.5 seconds. (a) What is the common angular acceleration of the carts? (b) What is the tangential acceleration of the inner cart? (c) What is the tangential acceleration of the outer cart?

(a) \( \text{rad/s}^2 \)
(b) \( \text{m/s}^2 \)
(c) \( \text{m/s}^2 \)

12.3 A car starts a race from rest on a circular track and has a tangential speed of 43 m/s at the end of the third lap. The track has a radius of 91 m. If it has constant angular acceleration, what is the magnitude of its tangential acceleration?

\( \text{m/s}^2 \)

Section 14 - Tangential and centripetal acceleration

14.1 A race car drives around a circular track of radius 240 m at a constant tangential speed of 64 m/s. (a) What is the magnitude of the car's total acceleration? (b) What angle does the total acceleration vector make with the centripetal acceleration vector?

(a) \( \text{m/s}^2 \)
(b) \( ^\circ \)

14.2 A windmill starts from rest and rotates with a constant angular acceleration of 0.25 rad/s\(^2\). How many seconds after starting will the magnitude of the tangential acceleration of the tip of a blade equal the magnitude of the centripetal acceleration at the same point?

\( \text{s} \)

14.3 A car starts from rest and drives around a circular track with a radius of 45.0 m at constant tangential acceleration. If the car takes 27.0 s in its first lap around the track, (a) what is the magnitude of its overall acceleration at the end of that lap? (b) What angle does the acceleration vector make with a radius line?

(a) \( \text{m/s}^2 \)
(b) \( ^\circ \)

Section 16 - Interactive summary problem: 11.6 seconds to liftoff

16.1 Use the information given in the interactive problem in this section to calculate (a) the angular acceleration and (b) the distance of the rocket from the pivot that is required for the rocket to lift off in the stated time after exactly 2 revolutions. Express your answer to the nearest half meter. Test your answer with the simulation.

(a) \( \text{rad/s}^2 \)
(b) \( \text{m} \)

Additional Problems

A.1 A pulsar is a rapidly rotating neutron star. The Crab Pulsar is located in the Crab Nebula in the constellation Taurus. The pulsar is in the center of the close-up view of the nebula shown in the photograph. The periods of pulsars can be measured with great accuracy: The Crab Pulsar has a period of 0.033 s. (a) Find the pulsar's angular velocity. (b) The radius of the pulsar is estimated to be 10 km. Find the tangential velocity of a point on its equator.

(a) \( \text{rad/s} \)
(b) \( \text{m/s} \)

A.2 You are designing an uninhabited combat air vehicle (UCAV) that will be capable of making a 20 "gee" turn. That is, the magnitude of the centripetal acceleration during the turn can be as great as 20.0 times 9.80 m/s\(^2\). Assume that your UCAV flies at a speed of 331 m/s ("Mach 1") and that its mass is 5.00\( \times 10^3 \) kg. (a) What is the minimum radius of a horizontal turn that your UCAV can make? (b) What is the force ("thrust") in the horizontal direction that must be provided to make that turn?

(a) \( \text{m} \)
(b) \( \text{N} \)
A.3 A carnival game is set up so that two thin disks of equal size are fixed to the same horizontal axle, rotating at 2.5 rad/s. The disks are rotating above a frictionless table, and their rims just brush against the table. Each disk has a rectangular hole notched in it at the rim, and the holes have an angular separation of 0.25 rad as viewed down the length of the axle. The spacing between the disks, along the axis, is 0.11 m. In order to win the game, you must slide a puck along the table, through both holes. The holes are just big enough for the puck to get through, and if the puck hits a disk, you lose. What is the speed a puck must travel so that it slides through both holes?

\[ \text{__________ m/s} \]

A.4 A diver performs a dive starting from a handstand off a 10.0-meter platform. What is her average angular velocity during the dive if she completes exactly 3 revolutions before she hits the water?

\[ \text{__________ rad/s} \]

A.5 John and Joan walk in opposite directions around a circular path, starting from the same point. The path has a radius of 50.0 meters. John walks at 1.00 m/s, Joan at 1.25 m/s. How long will it take for them to meet?

\[ \text{__________ s} \]

A.6 Prove that for an object in uniform circular motion, the centripetal acceleration equals the radius times the square of the angular velocity.